Risk Aversion, Risk Premia, and the Labor Margin with Generalized Recursive Preferences

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Abstract

A flexible labor margin allows households to absorb shocks to asset values with changes in hours worked as well as changes in consumption. This ability to absorb shocks along both margins greatly alters the household's attitudes toward risk, as shown by Swanson (2012). In the present paper, I extend that analysis to the case of generalized recursive preferences, as in Epstein and Zin (1989) and Weil (1989), including multiplier preferences, as in Hansen and Sargent (2001). Understanding risk aversion for these preferences is especially important because they are the primary mechanism being used to bring macroeconomic models into closer agreement with asset prices. Measures of risk aversion commonly used in the literature—including traditional, fixed-labor measures and Cobb-Douglas composite-good measures—show no stable relationship to the equity premium in a standard macroeconomic model, while the closed-form expressions I derive here match the equity premium closely. Thus, taking into account the household's labor margin is necessary for risk aversion to correspond to asset prices in the model.

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1. Introduction

A growing macro-finance literature focuses on bringing standard macroeconomic models into better agreement with basic asset pricing facts, such as the equity premium.[1] In asset pricing models, a crucial parameter is risk aversion, the compensation that households require to hold a risky monetary payoff. At the same time, a key feature of standard macroeconomic models is that households have some ability to vary their labor supply. A fundamental difficulty with this line of research, then, is that much of what is known about risk aversion has been derived under the assumption that household labor is exogenously fixed.[2]

Swanson (2012) addresses this problem when households have standard expected utility preferences. In the present paper, I extend that analysis to generalized recursive preferences, as in Epstein and Zin (1989) and Weil (1989), including multiplier preferences, as in Hansen and Sargent (2001) and Strzalecki (2011). These preferences are a primary mechanism being used to bring macroeconomic models into better agreement with asset prices, so understanding risk aversion in this framework is very important for the macro-finance literature.[3] In fact, there is no conventional wisdom as to what the formula for risk aversion should be for these preferences when labor supply can vary, with different authors using different, somewhat ad hoc generalizations of the traditional, fixed-labor measure. In the present paper, I undertake a systematic and rigorous analysis of this important question.

Intuitively, a flexible labor margin allows households to absorb shocks to asset values with changes in hours worked as well as changes in consumption, which can greatly alter the household's attitudes toward risk. For example, with expected utility and period utility function $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l_t$, the quantity $-c\, u_{11}/u_1 = \gamma$ is often referred to as the household's coefficient of relative risk aversion, but in fact the household is *risk neutral* with respect to gambles over asset values or wealth (Swanson, 2012). Intuitively, the household is indifferent at the margin between using labor or consumption to absorb a shock to asset values, and the household in this example

[1] For example, Boldrin, Christiano, and Fisher (2001), Tallarini (2000), Rudebusch and Swanson (2008, 2012), Uhlig (2007), Van Binsbergen et al. (2012), Backus, Routledge, and Zin (2008), Gourio (2012, 2013), Palomino (2012), Andreasen (2012, 2013), Colacito and Croce (2012), Dew-Becker (2012), and Kung (2012) all consider asset pricing in dynamic macroeconomic models with a variable labor margin.

[2] For example, Arrow (1965) and Pratt (1964) define absolute and relative risk aversion, $-u''(c)/u'(c)$ and $-c\,u''(c)/u'(c)$, in a static model with a single consumption good. Similarly, Epstein and Zin (1989) and Weil (1989) define risk aversion for generalized recursive preferences in a dynamic model without labor (or, equivalently, in which labor is fixed).

[3] The vast majority of studies cited in the first footnote take this approach, the exceptions being Boldrin et al. (2001), Rudebusch and Swanson (2008), and Palomino (2012). One of the main advantages of generalized recursive preferences is that they allow risk aversion to be modeled independently from the household's other preference parameters, such as the intertemporal elasticity of substitution.

is clearly risk neutral with respect to gambles over hours.[4] In the present paper, I rigorously derive closed-form expressions for risk aversion in dynamic equilibrium models with generalized recursive preferences and arbitrary period utility function u, taking into account the effects of the household's flexible labor margin, and show that those effects can be significant.

I also show, theoretically and numerically, that risk premia on assets in a macroeconomic model are unrelated to traditional, fixed-labor measures of risk aversion unless labor is, in fact, fixed. By contrast, the closed-form expressions for risk aversion I derive here match risk premia in a standard (flexible-labor) real business cycle model closely. Thus, taking the household's labor margin into account is necessary for there to be a stable relationship between risk aversion and risk premia in the model.

Empirically, several previous studies find that households vary their labor supply in response to portfolio shocks. Imbens, Rubin, and Sacerdote (2001) show that individuals who win a prize in the lottery reduce their labor supply significantly. Coile and Levine (2009) document that older individuals are less likely to retire after the stock market performs poorly, and Coronado and Perozek (2003) find that households retire earlier when the stock market performs well. More generally, Pencavel (1986) and Killingsworth and Heckman (1986) survey estimates of the wealth effect on labor supply and find it to be significantly negative.

Theoretically, there are a few previous studies that extend the Arrow-Pratt definition of risk aversion beyond the one-good, one-period case, as in the present paper. I defer a detailed discussion of these studies until Section 3, below, but briefly highlight some of the key contributions here. Kihlstrom and Mirman (1974) discuss some of the difficulties involved in generalizing the Arrow-Pratt definition to multiple goods. Stiglitz (1969) measures risk aversion in a static (one-period), multiple-good setting using the household's indirect utility function rather than utility itself, essentially a special case of Proposition 1 of the present paper. Constantinides (1990) measures risk aversion in a dynamic economy with fixed labor using the household's value function, another special case of Proposition 1. Boldrin, Christiano, and Fisher (1997) apply Constantinides' definition to some very simple endowment economy models for which they can compute closed-form expressions for the value function, and hence risk aversion. Uhlig (2007) points out that when households have generalized recursive preferences, leisure affects asset prices because the value function V appears in the household's stochastic discount factor, and V depends on leisure. My analysis here differs from this previous work by deriving closed-form solutions for risk

[4] More generally, when $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l_t^{1+\chi}/(1+\chi)$, risk aversion equals $(\gamma^{-1} + \chi^{-1})^{-1}$, a combination of the parameters on the household's consumption and labor margins, reflecting the fact that the household absorbs shocks along both margins.

aversion in dynamic equilibrium models in general, demonstrating the importance of the labor margin, and showing the tight link between risk aversion and asset prices in these models.

The remainder of the paper proceeds as follows. Section 2 defines the dynamic equilibrium framework used in the analysis. Section 3 derives closed-form expressions for risk aversion in the model. Section 4 demonstrates the close connection between risk aversion and Lucas-Breeden asset prices in the model, both theoretically and with numerical examples. Section 5 verifies the accuracy of the closed-form expressions for risk aversion using numerical methods. Section 6 extends the results to the case of balanced growth. Section 7 provides the corresponding expressions for the case of multiplier preferences. Section 8 discusses some general implications and concludes. An Appendix provides details of proofs and numerical solutions that are outlined in the main text.

2. Dynamic Equilibrium Framework

2.1 Generalized Recursive Preferences and Value Function

Time is discrete and continues forever. At each time t, the household receives a utility flow $u(c_t, l_t)$, where $(c_t, l_t) \in \Omega \subseteq \mathbb{R}^2$ denotes the household's choice of consumption and hours worked in period t. I assume the period utility function u satisfies the following regularity conditions:

Assumption 1. *The function $u : \Omega \to \mathbb{R}$ is increasing in its first argument, decreasing in its second, twice-differentiable, and strictly concave.*

The household faces a flow budget constraint in each period,

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t, \tag{1}$$

and a no-Ponzi-scheme condition,

$$\lim_{T \to \infty} \prod_{\tau=t}^{T} (1 + r_{\tau+1})^{-1} a_{T+1} \geq 0, \tag{2}$$

where a_t denotes beginning-of-period assets and w_t, r_t, and d_t denote the real wage, real interest rate, and net transfer payments to the household, respectively. There is a finite-dimensional Markov state vector θ_t that is exogenous to the household and governs the processes for w_t, r_t, and d_t. Before choosing (c_t, l_t) in each period t, the household observes θ_t and hence w_t, r_t, and d_t. The state vector and information set of the household's optimization problem at each date t is thus $(a_t;\theta_t)$. Let X denote the domain of $(a_t;\theta_t)$, and $\Gamma\colon X\to\Omega$ describe the set-valued correspondence of feasible choices for (c_t,l_t) for each given $(a_t;\theta_t)$.

Let $(c^t,l^t)\equiv\{(c_\tau,l_\tau)\}_{\tau=t}^{\infty}$ denote a state-contingent plan for household consumption and labor from time t onward, where the explicit state-dependence of the plan is suppressed to reduce notation. Following Epstein and Zin (1989) and Weil (1989), the household has preferences over state-contingent plans ordered by the recursive functional

$$\widetilde{V}(c^t,l^t) \;=\; u(c_t,l_t)+\beta\left[E_t\,\widetilde{V}(c^{t+1},l^{t+1})^{1-\alpha}\right]^{1/(1-\alpha)}, \tag{3}$$

where $\beta\in(0,1)$, $\alpha\in\mathbb{R}$, E_t denotes the mathematical expectation conditional on the household's information set at time t, and (c^{t+1},l^{t+1}) denotes the state-contingent plan (c^t,l^t) from date $t+1$ forward.[5] Note that equation (3) has the same form as expected utility preferences, but with the expectation operator "twisted" and "untwisted" by the coefficient $1-\alpha$. When $\alpha=0$, (3) reduces to the special case of expected utility. When $\alpha\neq 0$, the intertemporal elasticity of substitution over deterministic consumption paths in (3) is the same as for expected utility, but the household's risk aversion with respect to gambles over future utility flows is amplified (or attenuated) by the additional curvature parameter α.

The household's "generalized value function" $V: X\to\mathbb{R}$ is defined to be the maximized value of (3), subject to the budget constraint (1)–(2). V satisfies the recursive equation

$$V(a_t;\theta_t) \;=\; \max_{(c_t,l_t)\in\Gamma(a_t;\theta_t)} u(c_t,l_t)+\beta\left(E_t\,V(a_{t+1};\theta_{t+1})^{1-\alpha}\right)^{1/(1-\alpha)}, \tag{4}$$

where a_{t+1} is given by (1). Technical conditions for the existence and uniqueness of V are discussed shortly.

Note that many authors use an alternative notation for the generalized value function,

$$U(a_t;\theta_t) \;=\; \max_{(c_t,l_t)\in\Gamma(a_t;\theta_t)} \left[\widetilde{u}(c_t,l_t)^{\rho}+\beta\left(E_t\,U(a_{t+1};\theta_{t+1})^{\widetilde{\alpha}}\right)^{\rho/\widetilde{\alpha}}\right]^{1/\rho}, \tag{5}$$

where $\rho\in\mathbb{R}$, $\rho<1$. This specification follows Epstein and Zin's (1989) original notation more closely, where those authors take $\widetilde{u}(c_t,l_t)=c_t$ in their framework without labor. However, setting $V=U^{\rho}$, $u=\widetilde{u}^{\rho}$, and $\alpha=1-\widetilde{\alpha}/\rho$, this can be seen to correspond exactly to (4).[6] Moreover, (4) has a much clearer relationship than (5) to standard dynamic programming results, regularity

[5] The case $\alpha=1$ is understood to correspond to $\widetilde{V}(c^t,l^t) \;=\; u(c_t,l_t)+\beta\exp\left[E_t\log\widetilde{V}(c^{t+1},l^{t+1})\right]$.

[6] For the case $\rho<0$, set $V=-U^{\rho}$ and $u=-\widetilde{u}^{\rho}$. The case $\rho=0$ corresponds to multiplier preferences, which I consider in Section 7, below.

conditions, and first-order conditions: for example, (4) requires concavity of u while (5) requires concavity of $\widetilde{u}^\rho$, and the Benveniste-Scheinkman equation for (4) is the usual $V_1 = (1+r_t)u_1$ rather than $U_1 = (1+r_t)U^{(1-\rho)/\rho}\widetilde{u}^{\rho-1}\widetilde{u}_1$. That is, the marginal value of wealth in (4) is just the usual marginal utility of consumption rather than something much more complicated.

I require a few technical conditions to ensure that (3)–(4) are well-defined. First, to avoid complex numbers:

Assumption 2. *Either $u\colon \Omega \to [0,\infty)$ or $u\colon \Omega \to (-\infty, 0]$.*

In the latter case, it is natural to take $\widetilde{V} \leq 0$, $V \leq 0$, and interpret (3) as

$$\widetilde{V}(c^t, l^t) \;=\; u(c_t, l_t) - \beta \left[E_t(-\widetilde{V}(c^{t+1}, l^{t+1}))^{1-\alpha} \right]^{1/(1-\alpha)}, \tag{3$'$}$$

and similarly for (4). Note that (5) requires this same restriction, for the same reasons.[7]

Technical conditions that ensure the existence and uniqueness of V are tangential to the main points of the present paper, so for simplicity I assume:[8]

Assumption 3. *A solution $V\colon X \to \mathbb{R}$ to the household's generalized Bellman equation* (4) *exists and is unique, continuous, and concave.*

The same technical conditions, plus Assumption 1, guarantee the existence of a unique optimal choice for (c_t, l_t) at each point in time, given $(a_t;\theta_t)$. Let $c_t^* \equiv c^*(a_t;\theta_t)$ and $l_t^* \equiv l^*(a_t;\theta_t)$ denote the household's optimal choices of c_t and l_t as functions of the state $(a_t;\theta_t)$. To avoid boundary solutions for c_t^* and l_t^* that would make derivatives and first-order conditions problematic, I require these solutions to be interior:

Assumption 4. *For any $(a_t;\theta_t) \in X$, the household's optimal choice (c_t^*, l_t^*) exists, is unique, and lies in the interior of $\Gamma(a_t;\theta_t)$.*

[7] The assumption that either $u \geq 0$ or $u \leq 0$ is not very restrictive in practice. For example, restrictions can be placed on Ω or Γ and a constant added to u such that u never takes on negative (or positive) values. Alternatively, for local analysis around a steady state, the restriction is satisfied so long as $u \neq 0$ in steady state, since then $u \geq 0$ or $u \leq 0$ holds locally. Note that Assumption 2 is not required for multiplier preferences; see Section 7.

[8] Stokey and Lucas (1989), Alvarez and Stokey (1998), and Rincón-Zapatero and Rodríguez-Palmero (2003) provide different sets of sufficient conditions that ensure Assumption 3 is satisfied for the case $\alpha = 0$. Sufficient conditions for general α and general period utility functions $u(c_t, l_t)$ have not yet been derived in the literature, but there are proofs for important special cases. For example, Epstein and Zin (1989) prove the existence and uniqueness of V for general α when there is a single consumption good and no labor, which also applies here if consumption and leisure form an aggregate good, such as when $u(c_t, l_t)$ is Cobb-Douglas or CES. (In this case, we must include the household's "full labor income" in its budget constraint, but Epstein and Zin (1991) show how this can be handled within their framework.) Similarly, the results of Marinacci and Montrucchio (2010) for the case of a single consumption good also apply here when consumption and leisure form an aggregate good.

Intuitively, Assumption 4 requires the partial derivatives of u to grow sufficiently large toward the boundary that only interior solutions for c_t^* and l_t^* are optimal for any $(a_t;\theta_t) \in X$.[9]

It follows that V can be written as

$$V(a_t;\theta_t) \;=\; u(c_t^*, l_t^*) + \beta \left(E_t\, V(a_{t+1}^*;\theta_{t+1})^{1-\alpha}\right)^{1/(1-\alpha)}, \tag{6}$$

where $a_{t+1}^* \equiv (1+r_t)a_t + w_t l_t^* + d_t - c_t^*$.

Assumptions 1–4 guarantee that V is continuously differentiable with respect to a and satisfies the Benveniste-Scheinkman equation, but I will require slightly more than this below:

Assumption 5. *For any $(a_t;\theta_t)$ in the interior of X, the second derivative of V with respect to its first argument, $V_{11}(a_t;\theta_t)$, exists.*

Assumption 5 also implies differentiability of the optimal policy functions, c^* and l^*, with respect to a_t. Santos (1991) provides relatively mild sufficient conditions for this assumption to be satisfied when $\alpha = 0$; intuitively, u must be strongly concave.

2.2 Representative Household and Steady State Assumptions

Up to this point, the analysis has focused on a single household in isolation, leaving the other households of the model and the production side of the economy unspecified. Implicitly, the other households and production sector jointly determine the process for θ_t (and hence w_t, r_t, and d_t), and much of the analysis below does not need to be any more specific about these processes than this. However, to move from general expressions for risk aversion to more concrete, closed-form expressions, I adopt three standard assumptions from the macroeconomics literature:[10]

Assumption 6. *The household is* infinitesimal.

Assumption 7. *The household is* representative.

Assumption 8. *The model has a* nonstochastic steady state, *at which $x_t = x_{t+k}$ for all $k = 1, 2, \ldots$, and $x \in \{c, l, a, w, r, d, \theta\}$.*

Assumption 6 implies that an individual household's choices for c_t and l_t have no effect on the aggregate quantities w_t, r_t, d_t, and θ_t. Assumption 7 implies that, when the economy is at the

[9] Interiority is a standard requirement for differentiability of the household's optimal policy functions—see, e.g., Santos (1991). There are two main ways this assumption can be imposed: through Inada-type conditions on the period utility function u, or through restrictions on the domain X of the state variables a_t and θ_t. For example, if the household's period utility function is Cobb-Douglas over consumption and leisure, $u(c_t, l_t) = c_t^{\chi}(1-l_t)^{1-\chi}$, then the household's optimal choices of c_t and $1 - l_t$ will be strictly positive by the Inada-type conditions. To ensure $l_t > 0$ as well, the domain X must exclude cases where the household is so wealthy or wages are so low that the household would want to choose negative labor supply.

[10] Alternative assumptions about the nature of the other households in the model or the production sector may also allow for closed-form expressions for risk aversion. However, the assumptions used here are the most standard.

nonstochastic steady state, any individual household finds it optimal to choose the steady-state values of c and l given a and θ. Throughout the text, a variable without its time subscript t is used to denote its steady-state value.[11]

It is important to note that Assumptions 7–8 do not prohibit offering an individual household a hypothetical gamble of the type described below. The steady state of the model serves only as a reference point around which the *aggregate* variables w, r, d, and θ and the *other households'* choices of c, l, and a can be predicted with certainty. This reference point is important because it is there that closed-form expressions for risk aversion can be computed.

Finally, many dynamic models do not have a steady state *per se,* but rather a balanced growth path, as in King, Plosser, and Rebelo (1988). The results below carry through essentially unchanged to the case of balanced growth. For ease of exposition, Sections 3–5 restrict attention to the case of a steady state, while Section 6 shows the adjustments required under the more general:

Assumption 8′. *The model has a* balanced growth path *that can be renormalized to a nonstochastic steady state after a suitable change of variables.*

3. Risk Aversion

3.1 The Coefficient of Absolute Risk Aversion

The household's attitudes toward risk at time t generally depend on the household's state vector at time t, $(a_t;\theta_t)$. Given this state, I consider the household's aversion to a hypothetical one-shot gamble in period t of the form

$$a_{t+1} = (1+r_t)a_t + w_t l_t + d_t - c_t + \sigma\varepsilon_{t+1}, \tag{7}$$

where ε_{t+1} is a random variable representing the gamble, with bounded support $[\underline{\varepsilon}, \overline{\varepsilon}]$, mean zero, unit variance, independent of θ_τ for all times τ, and independent of a_τ, c_τ, and l_τ for all $\tau \leq t$. A few words about (7) are in order: First, the gamble is dated $t+1$ to clarify that its outcome is not in the household's information set at time t. Second, c_t cannot be made the subject of the gamble without substantial modifications to the household's optimization problem, because c_t is a choice variable under control of the household at time t. However, (7) is clearly equivalent to a one-shot gamble over net transfers d_t or asset returns r_t, both of which are exogenous to the

[11] Let the exogenous state θ_t contain the variances of any shocks to the model, so that $(a;\theta)$ denotes the nonstochastic steady state, with the variances of any shocks (other than the hypothetical gamble described in the next section) set equal to zero; $c(a;\theta)$ corresponds to the household's optimal consumption choice at the nonstochastic steady state, etc.

household.[12] Indeed, thinking of the gamble as being over r_t helps to illuminate the connection between (7) and the price of risky assets, which I will discuss further in Section 4. As shown there, the household's aversion to the gamble in (7) is directly linked to the premium households require to hold risky assets.

Following Arrow (1965) and Pratt (1964), I ask what one-time fee μ the household would be willing to pay in period t to avoid the gamble in (7):

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu. \tag{8}$$

Again following Arrow and Pratt, I define the quantity μ that makes the household just indifferent between (7) and (8), for infinitesimal σ and μ, to be the household's coefficient of absolute risk aversion. (I discuss the relationship of this definition to the broader literature shortly.) Formally, this corresponds to the following:

Definition 1. *Let $(a_t; \theta_t)$ be an interior point of X. Let $\hat{V}(a_t; \theta_t; \sigma)$ denote the value function for the household's optimization problem inclusive of the one-shot gamble (7), and let $\mu(a_t; \theta_t; \sigma)$ denote the value of μ that satisfies $V(a_t - \frac{\mu}{1+r_t}; \theta_t) = \hat{V}(a_t; \theta_t; \sigma)$. The household's* coefficient of absolute risk aversion at *$(a_t; \theta_t)$, denoted $R^a(a_t; \theta_t)$, is given by $R^a(a_t; \theta_t) = \lim_{\sigma \to 0} \mu(a_t; \theta_t; \sigma)/(\sigma^2/2)$.*

In Definition 1, $\mu(a_t; \theta_t; \sigma)$ denotes the household's "willingness to pay" to avoid a one-shot gamble of size σ in (7). As in Arrow (1965) and Pratt (1964), R^a denotes the limit of the household's willingness to pay per unit of variance as this variance becomes small. Note that $R^a(a_t; \theta_t)$ depends on the economic state because $\mu(a_t; \theta_t; \sigma)$ depends on that state. Proposition 1 shows that $\hat{V}(a_t; \theta_t; \sigma)$, $\mu(a_t; \theta_t; \sigma)$, and $R^a(a_t; \theta_t)$ in Definition 1 are well-defined and derives the expression for $R^a(a_t; \theta_t)$:

Proposition 1. *Let $(a_t; \theta_t)$ be an interior point of X. Given Assumptions 1–6, $\hat{V}(a_t; \theta_t; \sigma)$, $\mu(a_t; \theta_t; \sigma)$, and $R^a(a_t; \theta_t)$ exist and*

$$R^a(a_t; \theta_t) \;=\; \frac{-E_t \left[V(a^*_{t+1}; \theta_{t+1})^{-\alpha} V_{11}(a^*_{t+1}; \theta_{t+1}) \;-\; \alpha\, V(a^*_{t+1}; \theta_{t+1})^{-\alpha-1} V_1(a^*_{t+1}; \theta_{t+1})^2 \right]}{E_t\, V(a^*_{t+1}; \theta_{t+1})^{-\alpha} V_1(a^*_{t+1}; \theta_{t+1})}, \tag{9}$$

where V_1 and V_{11} denote the first and second partial derivatives of V with respect to its first argument. Given Assumptions 7–8, equation (9) can be evaluated at the steady state to yield:

$$R^a(a; \theta) \;=\; \frac{-V_{11}(a; \theta)}{V_1(a; \theta)} \;+\; \alpha\, \frac{V_1(a; \theta)}{V(a; \theta)}\,. \tag{10}$$

[12] In this case, the realized transfer $d_t + \sigma\varepsilon_{t+1}$, or asset return $r_t + \sigma\varepsilon_{t+1}$, would not be in the household's time-t information set, $(a_t; \theta_t)$.

PROOF: See Appendix.[13]

Equation (10) can be decomposed into the sum of two components: the first term on the right-hand side is essentially *intra*temporal and does not depend on the parameter α, while the second term captures the household's additional aversion to risky utility flows in the *future* and is closely related to α. For $u, V \geq 0$, larger values of α imply higher levels of risk aversion.[14] For $u, V \leq 0$, the opposite is true: *lower* values of α (especially negative values) imply higher levels of risk aversion.

3.1.1 Relationship to the Literature

Definition 1 and Proposition 1 imply that risk aversion is related to the curvature of the value function V with respect to wealth rather than the curvature of period utility u with respect to consumption. Because the value function depends on the economic state θ_t, this implies that risk aversion, as defined above, also depends on the economic state. Here I discuss this issue and its relationship to the literature.

First, note that it is common to define risk aversion in terms of a fair gamble over money or wealth, as in Definition 1. Both Arrow (1965) and Pratt (1964) use this definition, working with a "utility function for money" (Pratt, 1964, p. 122) or wealth (Arrow, 1965).[15] Athough those authors do not explicitly specify the presence of multiple goods, to the extent that they intended their "utility function for money" to be an indirect rather than a direct utility function, their measure of risk aversion depends on the economic environment (such as prices), just as in the present paper. Stiglitz (1969) follows the same approach and is much more explicit about measuring risk aversion using the indirect utility function U: he defines absolute and relative risk aversion, $-U_{yy}/U_y$ and $-yU_{yy}/U_y$, where "U is a function of both [income] y and [prices] p: $U = U(y, p)$. Arrow and Pratt assumed in effect that p is constant" (Stiglitz, 1969, p. 663 and footnote 11).

[13] When generalized recursive preferences are written in the form (5), the corresponding expressions are

$$R^a(a_t; \theta_t) = \frac{-E_t \left[U(a^*_{t+1}; \theta_{t+1})^{\widetilde{\alpha}-1} U_{11}(a^*_{t+1}; \theta_{t+1}) \; + \; (\widetilde{\alpha} - 1) U(a^*_{t+1}; \theta_{t+1})^{\widetilde{\alpha}-2} U_1(a^*_{t+1}; \theta_{t+1})^2 \right]}{E_t \, U(a^*_{t+1}; \theta_{t+1})^{\widetilde{\alpha}-1} U_1(a^*_{t+1}; \theta_{t+1})}$$

and

$$R^a(a; \theta) = \frac{-U_{11}(a; \theta)}{U_1(a; \theta)} \; + \; (1 - \widetilde{\alpha}) \frac{U_1(a; \theta)}{U(a; \theta)} \, .$$

[14] Sufficiently low or negative values of α can imply risk-loving behavior, $R^a(a; \theta) < 0$. The parameter α also determines the household's preference for early vs. late resolution of uncertainty, as discussed in Kreps and Porteus (1978) and Epstein and Zin (1989), because α determines the household's aversion to uncertainty about future utility flows V. For $u, V \geq 0$, the household prefers early resolution of uncertainty if $\alpha > 0$, and late resolution of uncertainty if $\alpha < 0$; for $u, V \leq 0$, the household prefers early resolution if $\alpha < 0$, and late resolution if $\alpha > 0$. These conditions correspond to the criterion $\widetilde{\alpha} < \rho$ in (5), emphasized by Epstein and Zin (1989).

[15] "Let Y = wealth, $U(Y)$ = total utility of wealth Y" (Arrow, 1965, pp. 91–92).

Thus, in Stiglitz (1969), risk aversion clearly depends on the economic environment—the price vector p—as well as the agent's preferences. Constantinides (1990) and Campbell and Cochrane (1999) work in a dynamic framework, where the household's aversion to money or wealth bets implies that it is the *value* function that is relevant, as in the present paper: "I define the RRA coefficient in terms of an atemporal gamble that changes the current level of capital by the outcome of the gamble... $RRA = -WV_{ww}/V_w$ ... The RRA coefficient is a function of wealth and of the state variables $x(t)$," (Constantinides, 1990, p. 527); and "Risk aversion measures attitudes toward pure wealth bets and is therefore conventionally captured by the second partial derivative of the value function with respect to individual wealth..." (Campbell and Cochrane, 1999, p. 243). The latter authors go on to discuss risk aversion in this way for two more pages, noting that risk aversion "depends on individual wealth W and on aggregate variables that describe asset prices or investment opportunities..." (Campbell and Cochrane, 1999, p. 244). In all of these papers, then, risk aversion depends on the economic environment (i.e., state variables, prices, etc.) as well as preferences.[16]

Thus, Definition 1 in the present paper is not unusual. Moreover, there is a considerable amount of "folk wisdom" in the literature that risk aversion in a dynamic environment *should* be measured using the value function V, and that this definition depends on the economic environment as well as on preferences. Swanson (2012), and Definition 1 and Proposition 1, above, formalize this folk wisdom and provide a rigorous derivation. This formalization is particularly important in the present paper, since there is no folk wisdom for what the formula for risk aversion should be in the case of generalized recursive preferences with multiple goods.

However, Kihlstrom and Mirman (1974, 1981) and some other authors promote an alternative view, in which a household's attitudes toward risk are defined solely as a function of its preferences and consumption bundle. According to this view, the household's attitudes toward risk are a property of its preferences (and consumption bundle) alone, and do not depend on its economic environment. If the household has preferences over multiple goods, then the household's attitudes toward risk are complicated and multi-dimensional, and cannot be summarized by a single "coefficient of risk aversion" except in very special cases. For example, in Kihlstrom and Mirman (1974), one agent cannot be said to be more or less risk averse than another unless both agents have identical ordinal preferences over all commodities. Similarly, in Kihlstrom and Mirman (1981), increasing, constant, and decreasing relative risk aversion are only defined if the

[16] See also Farmer (1990) and Flavin and Nakagawa (2008), who use essentially the same definition and come to the same conclusions in their dynamic frameworks.

agent has preferences that are homothetic.

According to this alternate, Kihlstrom-Mirman (1974, 1981) view, there generally is no single *coefficient* of risk aversion that can be defined as a function of preferences and the consumption bundle alone. However, when a single coefficient can be defined, such as when preferences are homothetic, then Kihlstrom and Mirman (1974, Section 3.1) show that this coefficient corresponds to the Stiglitz (et al.) definition above, based on monetary gambles. In some respects, then, the Stiglitz et al. coefficient of risk aversion can be viewed as a generalization of the Kihlstrom-Mirman coefficient, because the former is defined for a larger class of preferences than the latter, while the two definitions agree when the latter is defined. However, anything that affects the household's value function without altering preferences—such as a borrowing constraint—will affect the Stiglitz et al. coefficient of risk aversion and not the Kihlstrom-Mirman coefficient. In this respect, then, the Stiglitz et al. coefficient is not a generalization, because even when preferences are homothetic, the two coefficients may not agree due to borrowing constraints or other features of the economic environment that affect one coefficient and not the other.

The definitions and results in the present paper apply to the monetary, Arrow-Pratt-Stiglitz-Constantinides-Campbell-Cochrane definition of risk aversion described above and have essentially nothing to say about the alternative, Kihlstrom-Mirman definition (except when the two definitions agree). The reason for this focus is asset pricing: a household's aversion to a money or wealth bet is always well-defined under Assumptions 1–5 above and is highly relevant for understanding the household's aversion to holding a risky asset with monetary payoffs, such as a stock or bond.[17] The Kihlstrom-Mirman (1974, 1981) approach is multi-dimensional in general and provides little insight into how much compensation a household would require to hold a risky asset with monetary payoffs, except in very special cases such as homotheticity. Thus, the Kihlstrom-Mirman perspective does not give unambiguous answers about whether the risk premium on a given asset should be large or small, except when preferences are homothetic; in contrast, the Stiglitz et al. approach gives answers that are very useful, as I'll show below.

3.1.2 Absolute Risk Aversion

Returning to Definition 1 and Proposition 1, a practical difficulty with these expressions is that closed-form solutions for V do not exist in general, even for the simplest dynamic models with labor. We can solve this problem by observing that V_1 and V_{11} often can be computed even when

[17] The set of assets with monetary payoffs includes essentially all traded assets in the data, even "real" or "indexed" assets, since those assets' payoffs are transacted in money. Even in the case of a real asset such as an oil well, whose payoffs are in barrels of crude, those payoffs are typically converted into money at a spot rate.

closed-form solutions for V cannot be. For example, the Benveniste-Scheinkman equation,

$$V_1(a_t;\theta_t) = (1+r_t)\, u_1(c_t^*, l_t^*), \tag{11}$$

states that the marginal value of a dollar of assets equals the marginal utility of consumption times $1+r_t$ (the interest rate appears here because beginning-of-period assets in the model generate income in period t).[18] In (11), u_1 is a known function. Although closed-form solutions for the functions c^* and l^* are not known in general, the points c_t^* and l_t^* often are known—for example, when they are evaluated at the nonstochastic steady state, c and l. Thus, we can compute V_1 at the nonstochastic steady state by evaluating (11) at that point.

The second derivative V_{11} can be computed by noting that (11) holds for general a_t; hence we can differentiate it to yield:

$$V_{11}(a_t;\theta_t) \;=\; (1+r_t)\left[u_{11}(c_t^*, l_t^*)\,\frac{\partial c_t^*}{\partial a_t} \;+\; u_{12}(c_t^*, l_t^*)\,\frac{\partial l_t^*}{\partial a_t}\right]. \tag{12}$$

All that remains is to find the derivatives $\partial c_t^*/\partial a_t$ and $\partial l_t^*/\partial a_t$.

We solve for $\partial l_t^*/\partial a_t$ by differentiating the household's intratemporal optimality condition,

$$-u_2(c_t^*, l_t^*) \;=\; w_t\, u_1(c_t^*, l_t^*), \tag{13}$$

with respect to a_t, and rearranging terms to yield:

$$\frac{\partial l_t^*}{\partial a_t} = -\lambda_t \frac{\partial c_t^*}{\partial a_t}\,, \tag{14}$$

where

$$\lambda_t \;\equiv\; \frac{w_t u_{11}(c_t^*, l_t^*) + u_{12}(c_t^*, l_t^*)}{u_{22}(c_t^*, l_t^*) + w_t u_{12}(c_t^*, l_t^*)} \;=\; \frac{u_1(c_t^*, l_t^*)u_{12}(c_t^*, l_t^*) - u_2(c_t^*, l_t^*)u_{11}(c_t^*, l_t^*)}{u_1(c_t^*, l_t^*)u_{22}(c_t^*, l_t^*) - u_2(c_t^*, l_t^*)u_{12}(c_t^*, l_t^*)}\,. \tag{15}$$

Note that, if consumption and leisure in period t are normal goods, then $\lambda_t > 0$, although we do not require this restriction below. It now only remains to solve for the derivative $\partial c_t^*/\partial a_t$.

Intuitively, $\partial c_t^*/\partial a_t$ should not be too difficult to compute: it is just the household's marginal propensity to consume today out of a change in assets, which can be deduced from the household's Euler equation and budget constraint:[19]

[18] The Benveniste-Scheinkman equation (11) holds for generalized recursive preferences as well as expected utility. See the proof of Proposition 1 in the Appendix.

[19] The notation $\frac{\partial c_{t+1}^*}{\partial a_t}$ is taken to mean $\frac{\partial c_{t+1}^*}{\partial a_{t+1}}\frac{da_{t+1}^*}{da_t} = \frac{\partial c_{t+1}^*}{\partial a_{t+1}}\left[1 + r_{t+1} + w_t\frac{\partial l_t^*}{\partial a_t} - \frac{\partial c_t^*}{\partial a_t}\right]$, and analogously for $\frac{\partial c_{t+2}^*}{\partial a_t}$, $\frac{\partial c_{t+3}^*}{\partial a_t}$, etc.

Lemma 2. *Given Assumptions 1–8, the household's marginal propensity to consume out of wealth in a neighborhood of the nonstochastic steady state satisfies*

$$\frac{\partial c_t^*}{\partial a_t} = E_t \frac{\partial c_{t+1}^*}{\partial a_t} = E_t \frac{\partial c_{t+k}^*}{\partial a_t}, \qquad k = 1, 2, 3, \ldots, \tag{16}$$

and

$$\frac{\partial c_t^*}{\partial a_t} = \frac{r}{1 + w\lambda}. \tag{17}$$

PROOF: See Appendix.

In other words, starting near the nonstochastic steady state, the household's optimal change in consumption today in response to an increase in assets must be the same as the expected change in consumption tomorrow, and the expected change in consumption at each future date $t+k$. Note that this equality does not follow from the steady-state assumption—for example, in a model with internal habits, the individual household's optimal consumption response to a change in assets increases gradually over time, even starting from steady state.

According to Lemma 2, the household's optimal response to a unit increase in assets is to raise consumption in every period by the extra asset income, r (essentially the permanent income hypothesis), adjusted downward by the amount $1 + w\lambda$, which takes into account the household's decrease in hours worked and labor income. Thus, Lemma 2 represents a modified PIH that takes into account the household's labor margin.

We can now compute the household's coefficient of absolute risk aversion. Substituting (11), (12), (14), and (17) into (10) proves the following:

Proposition 3. *Given Assumptions 1–8, the household's coefficient of absolute risk aversion, $R^a(a_t; \theta_t)$, evaluated at steady state, satisfies*

$$R^a(a; \theta) = \frac{-u_{11} + \lambda u_{12}}{u_1} \frac{r}{1 + w\lambda} + \alpha \frac{r\, u_1}{u}, \tag{18}$$

where u_1, u_{11}, and u_{12} denote the corresponding partial derivatives of u evaluated at the steady state (c, l), and λ is given by (15) evaluated at steady state.

There are several features of Proposition 3 worth noting. First, when $\alpha = 0$, equation (18) reduces to the expressions derived in Swanson (2012) for the case of expected utility. When $\alpha = 0$ and labor is fixed ($\lambda = 0$), risk aversion in (18) reduces further to $-ru_{11}/u_1$, which is just the usual Arrow-Pratt definition multiplied by r, a scaling factor that translates between assets and current-period consumption units.[20]

[20] A gamble over a lump sum of $\$x$ is equivalent here to a gamble over an annuity of $\$x/r$. Thus, even though V_{11}/V_1 is different from u_{11}/u_1 by a factor of r, this difference is exactly the same as a change from lump-sum to annuity units. Thus, the difference in scale is essentially one of units.

When $u \geq 0$ everywhere, risk aversion is increasing in α, and when $u \leq 0$, R^a is decreasing in α, as observed after Proposition 1. Multiplying u by a constant has no effect on risk aversion, but an additive translation of u *does* affect risk aversion if $\alpha \neq 0$, because it changes the "twisted" expectation in equation (4). When $\alpha \neq 0$ and labor is fixed ($\lambda = 0$), equation (18) reduces to $\frac{-u_{11}}{u_1} + \alpha \frac{u_1}{u}$, times r, corresponding to the standard formula for absolute risk aversion in an Epstein-Zin-Weil endowment economy (see Example 1, below).[21]

When $\lambda \neq 0$, households can partially offset shocks to income through changes in hours worked. Even when consumption and labor are additively separable in u ($u_{12} = 0$), c_t^* and labor supply are indirectly connected through the household's budget constraint. When $u_{12} \neq 0$, risk aversion is further attenuated or amplified by the direct interaction between consumption and labor in utility, u_{12}. Note, however, that regardless of the signs of λ and u_{12}, R^a is always reduced when households can vary their labor supply:

Corollary 4.

$$R^a(a;\theta) \;\leq\; \frac{-ru_{11}}{u_1} \;+\; \alpha \frac{ru_1}{u}\,. \tag{19}$$

Note that the right-hand side of (19) is the formula for risk aversion with generalized recursive preferences when labor is exogenously fixed.

Proof: Substituting in for λ and w, the first term in (18) can be written as:

$$\frac{-ru_{11}}{u_1} \; \frac{u_{11}u_{22} - u_{12}^2}{u_{11}u_{22} - 2\frac{u_2}{u_1}\, u_{11}u_{12} + \left(\frac{u_2}{u_1}\right)^2 u_{11}^2} \;=\; \frac{-ru_{11}}{u_1} \; \frac{1}{1 + \dfrac{\left(\frac{u_2}{u_1}\, u_{11} - u_{12}\right)^2}{u_{11}u_{22} - u_{12}^2}}\,. \tag{20}$$

Strict concavity of u implies $u_{11}u_{22} - u_{12}^2 > 0$, hence the right-hand side of (20) is less than or equal to $-ru_{11}/u_1$.

The household's labor margin can have dramatic effects on risk aversion. Even if $-u_{11}/u_1$ is very large, the first term in (20) can be arbitrarily small as the discriminant, $u_{11}u_{22} - u_{12}^2$, approaches zero. In other words, the first term in (20) depends on the concavity of u in all dimensions rather than just in one dimension. The second term in (19)–(20), $\alpha r u_1/u$, is not directly affected by a change from a fixed-labor to flexible-labor assumption, however.

[21] When generalized recursive preferences are written in the form (5), $w = -\widetilde{u}_2/\widetilde{u}_1$, $\lambda = \dfrac{w\widetilde{u}_{11} + \widetilde{u}_{12}}{\widetilde{u}_{22} + w\widetilde{u}_{12}}$, and

$$R^a(a;\theta) = \left[\frac{-\widetilde{u}_{11} + \lambda \widetilde{u}_{12}}{\widetilde{u}_1} + (\rho - 1)\frac{-\widetilde{u}_1 + \lambda \widetilde{u}_2}{\widetilde{u}}\right] \frac{r}{1 + w\lambda} \;+\; (\rho - \widetilde{\alpha})\frac{r\,\widetilde{u}_1}{\widetilde{u}}\,.$$

This expression is somewhat more complicated than (18), owing to the more complicated derivatives of (5). When $\lambda = 0$ and $\widetilde{u} = c$, this reduces to $(1 - \widetilde{\alpha})/c$, the traditional fixed-labor measure of absolute risk aversion in Epstein and Zin (1989), Weil (1989), and Example 1.

I provide examples of risk aversion calculations in Section 3.3, below, after first defining relative risk aversion.

3.2 The Coefficient of Relative Risk Aversion

The distinction between absolute and relative risk aversion lies in the size of the hypothetical gamble faced by the household. If the household faces a one-shot gamble of size A_t in period t,

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + A_t \sigma \varepsilon_{t+1}, \tag{21}$$

or the household can pay a one-time fee $A_t \mu$ in period t to avoid this gamble, then it follows from Proposition 1 that $\lim_{\sigma \to 0} 2\mu(\sigma)/\sigma^2$ for this gamble is given by

$$A_t R^a(a_t; \theta_t). \tag{22}$$

The natural definition of A_t, considered by Arrow (1965) and Pratt (1964), is the household's wealth at time t. The gamble in (21) is then over a fraction of the household's wealth and (22) is referred to as the household's coefficient of relative risk aversion.

In models with labor, however, household wealth can be more difficult to define because of the presence of human capital. There are two natural definitions of human capital in these models, leading to two measures of household wealth A_t and hence two coefficients of relative risk aversion (22). Note that the household's financial assets a_t is *not* a good measure of wealth A_t, because a_t for an individual household may be zero or negative at some points in time.

When the household's time endowment is not well-defined, such as when $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l_t^{1+\chi}$ and no upper bound $\bar{l}$ on l_t is specified, or $\bar{l}$ is specified but is arbitrary, it is most natural to define human capital as the present discounted value of labor income, $w_t l_t^*$. Equivalently, total household wealth A_t equals the present discounted value of consumption, which follows from the budget constraint (1)–(2). This leads to the following definition:

Definition 2. *Let $(a_t; \theta_t)$ be an interior point of X. The household's* consumption-wealth coefficient of relative risk aversion, *denoted $R^c(a_t; \theta_t)$, is given by* (22) *with wealth $A_t = A_t^c \equiv (1 + r_t)^{-1} E_t \sum_{\tau=t}^{\infty} m_{t,\tau} c_\tau^*$, the present discounted value of household consumption, where $m_{t,\tau}$ denotes the stochastic discount factor $\prod_{s=t}^{\tau-1} m_{s+1}$, and m_{s+1} is given by* (37).

The factor $(1+r_t)^{-1}$ in the definition expresses wealth A_t^c in beginning- rather than end-of-period-t units, so that in steady state $A^c = c/r$ and $R^c(a;\theta)$ is given by

$$R^c(a;\theta) \;=\; \frac{-A^c\, V_{11}(a;\theta)}{V_1(a;\theta)} \;+\; \alpha\, \frac{A^c\, V_1(a;\theta)}{V(a;\theta)} \;=\; \frac{-u_{11} + \lambda u_{12}}{u_1}\, \frac{c}{1 + w\lambda} \;+\; \alpha\, \frac{c u_1}{u}\,. \tag{23}$$

Alternatively, when the household's time endowment $\bar{l}$ is well specified, I can define human capital to be the present discounted value of the household's time endowment, $w_t\bar{l}$. Equivalently, total household wealth A_t equals the present discounted value of leisure $w_t(\bar{l} - l_t^*)$ plus consumption c_t^*, from (1)–(2). This corresponds to the following definition:

Definition 3. *Let* $(a_t; \theta_t)$ *be an interior point of* X. *The household's* consumption-and-leisure-wealth coefficient of relative risk aversion, *denoted* $R^{cl}(a_t; \theta_t)$, *is given by* (22) *with wealth* $A_t = A_t^{cl} \equiv (1 + r_t)^{-1} E_t \sum_{\tau=t}^{\infty} m_{t,\tau} \big(c_\tau^* + w_\tau(\bar{l} - l_\tau^*)\big)$.

In steady state, $A^{cl} = \big(c + w(\bar{l} - l)\big)/r$, and

$$R^{cl}(a; \theta) \;=\; \frac{-u_{11} + \lambda u_{12}}{u_1} \, \frac{c + w(\bar{l} - l)}{1 + w\lambda} \;+\; \alpha \, \frac{\big(c + w(\bar{l} - l)\big) u_1}{u} \,. \tag{24}$$

Since leisure is positive, $R^c(a_t; \theta_t) < R^{cl}(a_t; \theta_t)$ because the size of the gamble is smaller. The closed-form expressions (23)–(24) are also closely related, differing only by the ratio of the two gambles, $(c + w(\bar{l} - l))/c$.[22]

For expositional purposes, I define

$$R^{fl}(a; \theta) \;\equiv\; \frac{-c\, u_{11}}{u_1} + \alpha\, \frac{c\, u_1}{u} \,, \tag{25}$$

the coefficient of relative risk aversion in the corresponding model where labor is held exogenously fixed (see Example 1, below). R^{fl} thus ignores the household's ability to offset shocks to portfolio values by varying labor supply. By Corollary 4, $R^c(a; \theta) \leq R^{fl}(a; \theta)$. However, $R^{cl}(a; \theta)$ may be greater or less than $R^{fl}(a; \theta)$, depending on the importance of leisure in the household's total consumption bundle.

3.3 Examples

Example 1. Following Epstein and Zin (1989) and Weil (1989), consider the case where utility depends only on consumption,

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1 - \gamma} \,, \tag{26}$$

with $\gamma > 0$, $c_t \geq 0$, and l_t fixed exogenously at some $l \in \mathbb{R}$ for all t.[23] Leisure is arbitrary in this example—any $\bar{l} > l$ is observationally equivalent—so R^{cl} from Definition 3 is not well-defined.

[22] Both Definitions 2 and 3 represent a proper generalization of the traditional definition of risk aversion in an endowment economy. First, both definitions reduce to R^{fl}, defined below, when there is no labor in the model. Second, in steady state the household consumes exactly the flow of income from its wealth, rA, consistent with standard permanent income theory (where one must include the value of leisure $w(\bar{l} - l)$ as part of consumption when the value of leisure is included in wealth).

[23] In this example, Assumptions 1–8 need to be modified in a straightforward way to the one-dimensional case.

Thus, attention is restricted to R^c from Definition 2,

$$R^c(a;\theta) \;=\; \frac{-c\,u_{11}}{u_1} \;+\; \alpha\,\frac{c\,u_1}{u} \;=\; \gamma+\alpha(1-\gamma)\,, \tag{27}$$

which motivates the definition of R^{fl} given above. Note that if the household's generalized value function is written using specification (5) rather than (4), with $\rho\equiv 1-\gamma$, then $1-\widetilde{\alpha}=\gamma+\alpha(1-\gamma)$ and $R^c(a;\theta) \;=\; 1-\widetilde{\alpha}$. This is the usual definition of risk aversion for generalized recursive preferences in an endowment economy.

Example 2. Following van Binsbergen et al. (2012), among others,[24] a natural way to incorporate leisure into the preferences in (26) is to let

$$u(c_t,l_t) = \frac{\big(c_t^{\chi}(1-l_t)^{1-\chi}\big)^{1-\gamma}}{1-\gamma}\,, \tag{28}$$

where $\gamma>0$, $\chi\in(0,1)$, $c_t>0$, and $l_t\in(0,1)$.[25] In this example, the household can be regarded as consuming a single, composite good in each period formed from the Cobb-Douglas aggregate of consumption and leisure. A natural definition of risk aversion is thus $\gamma+\alpha(1-\gamma)=1-\widetilde{\alpha}$, the coefficient of relative risk aversion from Example 1 applied to the single, composite good. Indeed, this is the definition used by van Binsbergen et al. (2012). It is also the value implied by Definition 3 of the present paper, which includes the value of leisure in household wealth:

$$R^{cl}(a;\theta) \;=\; \frac{-u_{11}+\lambda u_{12}}{u_1}\,\frac{c+w(1-l)}{1+w\lambda} \;+\; \alpha\,\frac{\big(c+w(1-l)\big)u_1}{u} \;=\; \gamma+\alpha(1-\gamma)\,. \tag{29}$$

The consumption-wealth coefficient of relative risk aversion from Definition 2, R^c, excludes leisure from household wealth and thus is smaller than (29), because the gamble is smaller:

$$R^c(a;\theta) \;=\; \frac{-u_{11}+\lambda u_{12}}{u_1}\,\frac{c}{1+w\lambda} \;+\; \alpha\,\frac{c\,u_1}{u} \;=\; \gamma\chi+\alpha(1-\gamma)\chi\,. \tag{30}$$

In this example, the Cobb-Douglas functional form implies $R^c(a;\theta)=\chi R^{cl}(a;\theta)$.[26] In the next section, I compare these two risk aversion measures to the risk premium on an equity security in a standard macroeconomic model, solved numerically.

Note that neither (29) nor (30) corresponds to the fixed-labor measure of risk aversion, $R^{fl}(a;\theta)=\frac{-c\,u_{11}}{u_1}+\alpha\,\frac{c\,u_1}{u}=\big(1-\chi(1-\gamma)\big)+\alpha(1-\gamma)$, a point emphasized by Swanson (2012) for

[24] See also Andreasen (2012, 2013), Gourio (2013), Colacito and Croce (2012), and Dew-Becker (2012).

[25] When $\gamma<1$, then $u>0$, risk aversion is increasing in α, and $\alpha>0$ corresponds to preferences that are more risk averse than expected utility. When $\gamma>1$, then $u<0$, risk aversion is decreasing in α, and $\alpha<0$ corresponds to preferences that are more risk averse than expected utility.

[26] That is, $c/(c+w(1-l))=\chi$. One might be surprised that $R^c(a;\theta)\to 0$ as $\chi\to 0$. However, as $\chi\to 0$, $w/c\to\infty$, so consumption becomes trivial to insure with tiny variations in labor supply.

the case of expected utility, $\alpha = 0$. The fixed-labor measure R^{fl} ignores the household's ability to offset shocks to portfolio values by varying its hours of work; as a result, R^{fl} does not generally correspond to the household's willingness to hold a risky asset and thus is not closely related to the equilbrium prices of such assets, as I will verify in the next section.

Finally, a number of other authors consider an alternative parameterization of (28),[27]

$$u(c_t, l_t) = \frac{\big(c_t(1-l_t)^{\nu}\big)^{1-\gamma}}{1-\gamma}, \tag{31}$$

where $\gamma > 0$, $\nu > 0$, $c_t > 0$, $l_t \in (0,1)$, and $\gamma > \nu/(1+\nu)$ for concavity. For this parameterization, $R^{fl}(a;\theta) = \gamma + \alpha(1-\gamma) = 1 - \widetilde{\alpha}$. However, recognizing the household's ability to self-insure portfolio fluctuations with changes in labor supply, we have:

$$R^{cl}(a;\theta) \;=\; \big(1 - (1-\gamma)(1+\nu)\big) \;+\; \alpha(1-\gamma)(1+\nu) \tag{32}$$

and

$$R^{c}(a;\theta) \;=\; \frac{1 - (1-\gamma)(1+\nu)}{1+\nu} \;+\; \alpha(1-\gamma)\,. \tag{33}$$

These expressions follow from Definitions 2 and 3 directly, or from the fact that (31) can be rewritten as $u(c_t, l_t) = \big(c_t^{1/(1+\nu)}(1-l_t)^{\nu/(1+\nu)}\big)^{(1-\gamma)(1+\nu)}$.

Example 3. Following Rudebusch and Swanson (2009) and many authors in the New Keynesian DSGE literature,[28] consider the additively separable period utility function

$$u(c_t, l_t) \;=\; \frac{c_t^{1-\gamma}}{1-\gamma} \;-\; \eta\frac{l_t^{1+\chi}}{1+\chi}, \tag{34}$$

where $\chi > 0$, $\eta > 0$, $c_t > 0$, $l_t > 0$, and $\gamma > 1$.[29] Leisure is essentially arbitrary in this example, since different assumptions regarding $\bar{l}$ have essentially no effect on the equilibrium. Thus, $R^{cl}(a;\theta)$ is not well-defined, so I restrict attention to $R^{c}(a;\theta)$ from Defintion 2,

$$R^{c}(a;\theta) \;=\; \frac{\gamma}{1 + \frac{\gamma}{\chi}\frac{wl}{c}} \;+\; \frac{\alpha(1-\gamma)}{1 + \frac{\gamma-1}{1+\chi}\frac{wl}{c}}\,. \tag{35}$$

As in Swanson (2012), we can simplify (35) a bit further by assuming $c \approx wl$, an assumption made in *this paragraph only* and nowhere else in the paper.[30] In this case,

$$R^{c}(a;\theta) \;\approx\; \frac{\gamma}{1 + \frac{\gamma}{\chi}} \;+\; \frac{\alpha(1-\gamma)}{1 + \frac{\gamma-1}{1+\chi}}\,. \tag{36}$$

[27] See Gourio (2012), Uhlig (2007), Backus, Routledge, and Zin (2008), and Kung (2012).

[28] See, e.g., Erceg, Henderson, and Levin (2000), Woodford (2003), Christiano, Eichenbaum, and Evans (2005), and Galí (2008). These New Keynesian DSGE studies do not use generalized recursive preferences, however.

[29] The last restriction ensures consistency with Assumption 2. Alternatively, one could assume restrictions on the domain Ω such that $u(\cdot,\cdot) < 0$ for all $(c_t, l_t) \in \Omega$. Under either of these assumptions, $u < 0$, risk aversion is decreasing in α, and $\alpha < 0$ corresponds to preferences that are more risk averse than expected utility.

[30] In steady state, $c = ra + wl + d$, so $c = wl$ holds exactly if there is neither capital nor transfers in the model. In any case, $ra + d$ is typically small, since $r \approx .01$.

Equation (36) is less than $R^{fl}(a;\theta) = \gamma + \alpha(1-\gamma)$, by an amount that can be dramatic if either of the denominators in (36) is large. On the other hand, as $\chi \to \infty$, the household's labor margin becomes inflexible and $R^c \to R^{fl}$.

4. Risk Aversion and Asset Pricing

So far, I have shown that the household's aversion to gambles over asset values or wealth depends on its ability to offset the outcome of those gambles with changes in hours worked. In this section, I extend the analysis to show the relationship between risk aversion and risk premia in the Lucas-Breeden stochastic discounting framework. Risk premia in this framework are closely related to the definition of risk aversion in the previous section, and are generally unrelated to traditional measures of risk aversion that hold household labor fixed.

4.1 The Stochastic Discount Factor, Risk Premia, and Risk Aversion

For generalized recursive preferences (4) with labor, Rudebusch and Swanson (2012) show that the household's stochastic discount factor is given by

$$m_{t+1} \;\equiv\; \beta\,\frac{u_1(c^*_{t+1}, l^*_{t+1})}{u_1(c^*_t, l^*_t)}\,\frac{V(a^*_{t+1};\theta_{t+1})^{-\alpha}}{\big(E_t V(a^*_{t+1};\theta_{t+1})^{1-\alpha}\big)^{-\alpha/(1-\alpha)}}\,. \tag{37}$$

Let p^i_t denote the ex-dividend time-t price of an asset i that pays stochastic dividend d^i_t each period. In equilibrium, p^i_t satisfies

$$p^i_t \;=\; E_t m_{t+1}(d^i_{t+1} + p^i_{t+1}). \tag{38}$$

Let $1 + r^i_{t+1}$ denote the realized gross return on the asset,

$$1 + r^i_{t+1} \;\equiv\; \frac{d^i_{t+1} + p^i_{t+1}}{p^i_t}\,, \tag{39}$$

and define the risk premium on the asset, ψ^i_t, to be its expected excess return,

$$\psi^i_t \;\equiv\; E_t r^i_{t+1} - r^f_{t+1}\,, \tag{40}$$

where $1 + r^f_{t+1} \equiv 1/E_t m_{t+1}$ denotes the risk-free rate. Then

$$\begin{aligned} \psi^i_t \;&=\; \frac{E_t m_{t+1} E_t(d^i_{t+1} + p^i_{t+1}) - E_t m_{t+1}(d^i_{t+1} + p^i_{t+1})}{p^i_t E_t m_{t+1}} \\ &=\; \frac{-\mathrm{Cov}_t(m_{t+1}, r^i_{t+1})}{E_t m_{t+1}}\,, \end{aligned} \tag{41}$$

where Cov_t denotes the covariance conditional on information at time t.

We can then relate the stochastic discount factor to risk aversion as follows.[31] First, the stochastic discount factor (37) can be differentiated at the nonstochastic steady state, conditional on information at time t, to yield

$$dm_{t+1} \;=\; \frac{\beta}{u_1}\left[u_{11}dc^*_{t+1} + u_{12}dl^*_{t+1}\right] \;-\; \frac{\alpha\beta}{V}\,dV_{t+1} \tag{42}$$

to first order. From the household's intratemporal optimality condition (13),

$$dl^*_{t+1} = -\lambda dc^*_{t+1} - \frac{u_1}{u_{22} + wu_{12}}\,dw_{t+1} \tag{43}$$

to first order. Note that there is an additional term in (43) relative to (14) because θ (and hence w, r, and d) will generally change in response to macroeconomic shocks.

The corresponding expression for dc^*_{t+1} is more complicated, so I state it as a lemma:

Lemma 5. *To first order in a neighborhood of the nonstochastic steady state,*

$$\begin{aligned} dc^*_{t+1} = \frac{r}{1+w\lambda}\left[da_{t+1} + E_{t+1}\sum_{k=1}^{\infty}\frac{1}{(1+r)^k}\left(l\,dw_{t+k} + dd_{t+k} + adr_{t+k}\right)\right] \\ + \frac{u_1u_{12}}{u_{11}u_{22} - u_{12}^2}\,dw_{t+1} \;+\; \frac{-u_1}{u_{11} - \lambda u_{12}}\,E_{t+1}\sum_{k=1}^{\infty}\frac{1}{(1+r)^k}\left(\frac{r\lambda}{1+w\lambda}\,dw_{t+k} - \beta dr_{t+k+1}\right). \end{aligned} \tag{44}$$

Proof: The expression follows from the household's Euler equation, budget constraint, and equation (43). See the Appendix for details.

[31] Intuitively, one can start to see the close relationship between the risk premium and risk aversion as follows: since $u_1(c^*_t, l^*_t) = V_1(a_t;\theta_t)/(1+r_t)$,

$$m_{t+1} \;=\; \beta\,\frac{V_1(a^*_{t+1};\theta_{t+1})}{V_1(a_t;\theta_t)}\;\frac{V(a^*_{t+1};\theta_{t+1})^{-\alpha}}{\left(E_tV(a^*_{t+1};\theta_{t+1})^{1-\alpha}\right)^{-\alpha/(1-\alpha)}}\;\frac{1+r_t}{1+r_{t+1}}\,.$$

Then, to first order around the nonstochastic steady state, conditional on information at time t,

$$\begin{aligned} dm_{t+1} \;&=\; \beta\,\frac{V_{11}da^*_{t+1} + V_{12}d\theta_{t+1}}{V_1} \;-\; \alpha\beta\,\frac{V_1da^*_{t+1} + V_2d\theta_{t+1}}{V} \;-\; \beta\,\frac{dr_{t+1}}{1+r} \\ &=\; -\beta R^a(a;\theta)\,da^*_{t+1} \;+\; \left(\frac{\beta V_{12}}{V_1} - \frac{\alpha\beta V_2}{V}\right)d\theta_{t+1} \;-\; \beta\,\frac{dr_{t+1}}{1+r}\,, \end{aligned}$$

assuming V is differentiable with respect to θ at the steady state, and where $dx_t \equiv x_t - x$, the time-t deviation of variable x from steady state. It follows that

$$\psi^i_t \;\approx\; R^a(a;\theta)\,\mathrm{Cov}_t(dr^i_{t+1}, da^*_{t+1}) \;+\; \left(\frac{-V_{12}}{V_1} + \alpha\,\frac{V_2}{V}\right)\mathrm{Cov}_t(dr^i_{t+1}, d\theta_{t+1}) \;+\; \beta\,\mathrm{Cov}_t(dr^i_{t+1}, dr_{t+1})$$

near the steady state. Here, ψ^i_t increases linearly with R^a, by an amount that depends on the covariance of the asset return with the household's financial wealth.

However, this decomposition is problematic for several reasons. First, the covariance involving da^*_{t+1} ignores the household's nonfinancial wealth, such as the present value of future transfers and labor income. Instead, the asset's covariance with nonfinancial wealth is relegated to the *second* term above, since θ determines the household's current and future wages w and transfers d. But this covariance is expressed in terms of the "black box" state variable θ rather than nonfinancial wealth itself, and the coefficient $(-V_{12}/V_1 + \alpha V_2/V)$ on this covariance is neither clearly related nor unrelated to risk aversion. Thus, the decomposition in the main text is more useful, albeit somewhat more complicated.

Note that if w, r, and d are held constant, as in the Arrow-Pratt gamble for a single household in Section 3, then equations (43)–(44) reduce to (14) and (17). More generally, (44) includes the effects of changes in w, r, and d on the household's desired consumption. The term in square brackets in (44) describes the change in household wealth—including nonfinancial wealth—and thus the first line of (44) describes the wealth effect on consumption. The last line of (44) describes the substitution effect: changes in consumption due to changes in current and future wages and interest rates.[32]

For notational simplicity, let $d\hat{A}_{t+1} \equiv da_{t+1} + E_{t+1} \sum_{k=1}^{\infty} (1+r)^{-k} (l\, dw_{t+k} + dd_{t+k} + a dr_{t+k})$, the change in household wealth in (44). Then it is straightforward to show:

Lemma 6. *To first order in a neighborhood of the nonstochastic steady state,*

$$dV_{t+1} \;=\; u_1 (1+r)\, d\hat{A}_{t+1}. \tag{45}$$

Proof: The expression follows from (6), (43), and (44). See the Appendix for details.

Lemma 6 states that the change in household welfare equals the marginal utility of consumption times the change in household wealth. The factor $1+r$ appears in (45) because a change in beginning-of-period-t assets produces $1+r$ units of extra consumption in period t.

Equations (42)–(45) then imply the following decomposition:

Proposition 7. *To first order in a neighborhood of the nonstochastic steady state,*

$$dm_{t+1} \;=\; -R^a(a;\theta)\, \beta\, d\hat{A}_{t+1} + \beta\, d\Phi_{t+1}\,, \tag{46}$$

where $d\Phi_{t+1} \equiv E_{t+1} \sum_{k=1}^{\infty} (1+r)^{-k} \big(\beta dr_{t+k+1} - \frac{r\lambda}{1+w\lambda}\, dw_{t+k}\big)$, *the intertemporal substitution term from* (44). *To second order in a neighborhood of the nonstochastic steady state,*

$$\psi_t^i \;=\; R^a(a;\theta)\, \mathrm{Cov}_t(dr_{t+1}^i, d\hat{A}_{t+1}) \;-\; \mathrm{Cov}_t(dr_{t+1}^i, d\Phi_{t+1})\,. \tag{47}$$

Proof: Substituting (43)–(45) into (42) yields (46). Substituting (46) into (41) yields (47). (Recall that $V = u/(1-\beta)$ and $\beta = E_t m_{t+1}$ at steady state.) Finally, $\mathrm{Cov}(dx, dy)$ is accurate to second order when dx and dy are accurate to first order.

The first term in the decomposition (47) shows that ψ_t^i increases locally linearly with R^a, by an amount that depends on the covariance between the asset return and the household's wealth, including nonfinancial wealth. This link between risk premia and risk aversion should not be

[32] The household's intertemporal elasticity of substitution is given by $-u_1/(c(u_{11} - \lambda u_{12}))$, so the last term in (44) describes intertemporal substitution effects on consumption of changes in future wages and interest rates.

too surprising: Propositions 1–2 described the risk premium for extremely simple, idiosyncratic gambles over household wealth, while Proposition 7 shows that the same coefficient also appears in the household's aversion to more general financial market gambles that may be correlated with aggregate variables such as interest rates, wages, and transfers.

The second term in (47) corresponds to Merton's (1973) "changes in investment opportunities" in the ICAPM framework. Even if $R^a = 0$—that is, even if households are risk-neutral for money or wealth bets—ψ_t^i can be nonzero. This is because even a risk-neutral household can benefit from an asset that pays off well when the price of the household's total consumption bundle is low. An asset that pays off well when current and future wages are low (and hence leisure is cheap) or current and future interest rates are high (and hence future consumption is cheap) is preferable to an asset that pays off poorly in those situations. Even a risk-neutral household would be willing to pay a premium for such an asset—implying a lower ψ_t^i—and this effect is captured by the second term in (47).

The fact that households in the present paper face a consumption-leisure tradeoff as well as a current-vs.-future consumption tradeoff implies that the second term in (47) is more general than just changes in the household's investment opportunities. Indeed, the second term in (47) is better described as being due to changes in *purchasing* opportunities—opportunities to purchase leisure and future consumption more cheaply. The decomposition in (47) also suggests that ψ_t^i is more accurately described as an "expected excess return" rather than a "risk premium" because only the first term in (47) represents compensation to the household for bearing risk; the second term is not compensation for risk but rather reflects the household's ability to use the asset to take advantage of changes in purchasing opportunities over time.

Finally, the decomposition (47) can be written in terms of relative rather than absolute risk aversion using Definitions 2–3:[33]

Corollary 8. *In terms of relative risk aversion, the risk premium in* (47) *can be written as:*

$$\psi_t^i \;=\; R^c(a;\theta)\,\mathrm{Cov}_t\!\left(dr_{t+1}^i, \frac{d\hat{A}_{t+1}}{A^c}\right) \;-\; \mathrm{Cov}_t(dr_{t+1}^i, d\Phi_{t+1}) \tag{48}$$

or

$$\psi_t^i \;=\; R^{cl}(a;\theta)\,\mathrm{Cov}_t\!\left(dr_{t+1}^i, \frac{d\hat{A}_{t+1}}{A^{cl}}\right) \;-\; \mathrm{Cov}_t(dr_{t+1}^i, d\Phi_{t+1})\,, \tag{49}$$

where A^c and A^{cl} are as in Definitions 2–3.

[33] Note that $d\hat{A}_{t+1}$ differs slightly from dA_{t+1}^c and dA_{t+1}^{cl}, which is why (48) and (49) are not written in terms of $d\log A_{t+1}^c$ or $d\log A_{t+1}^{cl}$.

4.2 Numerical Examples

Two numerical examples help to illustrate the relationship between risk aversion and risk premia derived above. For simplicity, the equity premium is studied in a standard real business cycle (RBC) framework, which provides just enough structure to create an interesting asset pricing problem in which household labor supply can vary endogenously.

The economy consists of a unit continuum of representative households and a unit continuum of perfectly competitive representative firms. Each household has optimization problem (1)–(4) and period utility function to be specified shortly. Each firm has production function

$$y_t = Z_t k_t^{1-\zeta} l_t^{\zeta}\,, \tag{50}$$

where y_t, k_t, and l_t denote firm output, beginning-of-period capital, and labor input, respectively. The productivity parameter Z_t is common across firms and follows the exogenous process

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_t, \tag{51}$$

where ε_t is i.i.d. with mean zero and variance σ_ε^2. Labor and capital are supplied by households at the competitive wage and rental rates w_t and r_t^k. Capital is the only asset in the economy that is in nonzero net supply. Households accumulate capital according to

$$k_{t+1} = (1 + r_t)k_t + w_t l_t - c_t, \tag{52}$$

where $r_t \equiv r_t^k - \delta$, δ is the capital depreciation rate, and c_t denotes household consumption.

For simplicity, define an equity security to be a claim on the aggregate consumption stream, where aggregate consumption $C_t = c_t$ in equilibrium. The ex-dividend price of the equity claim, p_t, then satisfies

$$p_t = E_t m_{t+1}(C_{t+1} + p_{t+1}) \tag{53}$$

in equilibrium, where m_{t+1} is given by (37). Let the equity premium, ψ_t, be defined as the expected excess return

$$\psi_t \;\equiv\; \frac{E_t(C_{t+1} + p_{t+1})}{p_t} \;-\; (1 + r_t^f)\,. \tag{54}$$

I follow standard calibrations in the literature and take a period in the model to be one quarter in the data, set β to .99, δ to .025, ζ to .7, and σ_ε to .01. I consider the cases $\rho_z < 1$ and $\rho_z = 1$ in the examples below. Once the period utility function is specified, I solve the model using perturbation methods, as in Rudebusch and Swanson (2012) and Swanson (2012). This involves computing a nonstochastic steady state for the model (or transformed version of the model) and an nth-order Taylor series approximation to the true nonlinear solution for the model's endogenous variables around the steady state.[34] Additional details of the solution algorithm and computer code are provided in the Appendix and in Swanson, Anderson, and Levin (2006). Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006) solve a standard RBC model using a variety of numerical methods and find that a fifth-order perturbation solution is among the most accurate methods globally as well as being faster to compute than other standard methods.

Example 4. Consider first the additively separable period utility function from Rudebusch and Swanson (2009) and Example 3,

$$u(c_t, l_t) \;=\; \frac{c_t^{1-\gamma}}{1-\gamma} \;-\; \eta \frac{l_t^{1+\chi}}{1+\chi}\,. \tag{55}$$

Set $\rho_z = 0.9$, $\gamma = 5$, $\chi = 1.5$, and $\alpha = -10$ as baseline values, and consider how the equity premium and risk aversion vary as each of γ, χ, and α are varied in turn.[35] For each set of parameter values, we can solve the model as described above.

[Figure 1 about here]

Figure 1 plots the equity premium and risk aversion as functions of χ, γ, and α. The solid black line in each panel graphs the equity premium, ψ, against the right axis. The equity premium in this model is very small, less than 25 basis points per year in each of the panels; this is a manifestation of Roewenhorst's (1995) and Lettau and Uhlig's (2000) finding that the equity premium is an even larger puzzle in RBC models with endogenous labor than in an endowment economy, because households can endogenously smooth consumption in response to shocks. The dashed blue line in each panel plots the coefficient of relative risk aversion, $R^c(a;\theta)$ from equation (35), against the left axis. For comparison, the dotted red line in each panel plots the fixed-labor measure of risk aversion for these preferences, $R^{fl}(a;\theta) = \gamma + \alpha(1-\gamma)$, also against the left axis.

In each of the three panels in Figure 1, the equity premium tracks R^c closely, and is essentially unrelated to R^{fl}. In the top panel, R^{fl} is independent of χ and thus is constant at 45, yet the equity premium varies by a factor of four, along with R^c. In the middle panel, R^{fl} increases

[34] Results in the figures below are for $n = 5$, a fifth-order approximation, but results are very similar for $n = 4$ and $n = 6$, suggesting that the Taylor series has essentially converged by $n = 5$ for the range of state variables considered in the figures.

[35] To allow for balanced growth or $\rho_z = 1$, the preference specification (55) would have to be modified, as in Rudebusch and Swanson (2012) and Swanson (2015). For simplicity, those modifications are not considered in this example.

linearly with γ, ranging from about 1 to 1090 (values above 32 are off the chart and not depicted), while the equity premium is a concave function of γ that corresponds closely to R^c. In the bottom panel, the equity premium varies about linearly with α and R^c, but does not correspond to R^{fl}.[36] Note that, in the bottom panel, more negative values of α imply greater risk aversion because $u \leq 0$; also, the equity premium does not converge to zero as $R^c \to 0$ due to the additional ICAPM term in (47) reflecting changes in purchasing opportunities discussed earlier.

[Figure 2 about here]

Intuitively, lower values of χ imply a more flexible labor margin, which gives the household more ability to insure itself from consumption fluctuations. This can be seen clearly in Figure 2, which plots first-order impulse response functions for consumption, labor, and the capital stock to a one percent positive shock to productivity Z_t. In each panel, the solid black line depicts the impulse response for the baseline parameterization of the model and the dashed and dotted lines plot impulse response functions for the cases $\chi = 5$ and $\chi = 0.1$, respectively. For all three parameterizations, consumption rises in response to the productivity shock, labor rises on impact and then falls, and household savings increases (as evidenced by the rise in the capital stock). When χ is lower, the household's labor margin is more flexible and the household reduces labor supply by more, on net, in response to the shock, thereby smoothing consumption. Note how this intuition holds despite the fact that labor initially *rises* on impact, as a result of the substitution effect on labor supply. Thus, the fact that the short-run correlation between labor and consumption is positive in the model does not prevent the household from using labor supply to smooth its consumption in response to shocks.

Example 5. Next, consider the Cobb-Douglas preference specification from van Binsbergen et al. (2012) and Example 2,

$$u(c_t, l_t) = \frac{\left(c_t^{\chi}(1-l_t)^{1-\chi}\right)^{1-\gamma}}{1-\gamma}. \tag{56}$$

Following Gourio (2013), set $\rho_z = 1$, $\gamma = 0.5$, $\chi = 0.3$, and $\alpha = 19$, and consider how the equity

[36] The equity premium ψ, R^c, and R^{fl} all vary about linearly with α, but the magnitude of R^{fl} does not agree with ψ. For example, in the top panel of Figure 1, an equity premium of about 14bp corresponds to risk aversion around 45 by either measure R^c or R^{fl}. In the bottom panel of Figure 1, ψ of about 15bp also corresponds to R^c of about 45 (at $\alpha \approx -27$), but would require $R^{fl} \approx 100$.

Results for the bond premium—the risk premium on a long-term real bond—are essentially the same as those for the equity premium, although the model-implied bond premium is generally smaller than the equity premium and can even be negative if interest rates tend to move countercylically, as discussed in Rudebusch and Swanson (2012). In this and the following example, the magnitude of the bond premium tracks R^c closely and does not correspond to R^{fl} or R^{cl}.

premium and risk aversion vary as χ, γ, and α are varied in turn.[37] For each set of parameter values, we can solve the model as described above.

[Figure 3 about here]

Figure 3 plots the equity premium and risk aversion as functions of χ, γ, and α. As in Figure 1, the solid black line in each panel depicts the equity premium, ψ, the dashed blue line plots the consumption-wealth coefficient of relative risk aversion, $R^c(a;\theta)$, and the dotted red line graphs the traditional, fixed-labor risk aversion measure, $R^{fl}(a;\theta)$. As in Figure 1, the equity premium in Figure 3 tracks R^c closely, and is essentially unrelated to R^{fl}. In the top panel, R^{fl} is nearly constant at a value of about 10, yet the equity premium varies by a factor of almost ten, along with R^c. (The equity premium does not quite converge to zero along with R^c due to the additional ICAPM term in (47) reflecting changes in purchasing opportunities.) In the middle panel, R^{fl} increases linearly as γ falls, ranging from about 1 to 19.5 (values above 12 are not depicted), but the equity premium increases at a more moderate pace corresponding to R^c. For example, a value of $\psi = 10$ bp is associated with $R^c \approx 5$ in the top panel of Figure 3, while a value of $\psi = 10$ bp in the middle panel requires $R^c \approx 5$ vs. $R^{fl} \approx 16$, at $\gamma \approx .2$. In the bottom panel, the equity premium increases about linearly with α and R^c, while R^{fl} again grows too quickly.

Household leisure is well-defined in this example, so we can also plot the consumption-and-leisure-wealth coefficient of relative risk aversion, $R^{cl}(a;\theta) = \gamma + \alpha(1-\gamma) = 1 - \widetilde{\alpha}$, as the dash-dotted green line in each panel of Figure 3. Perhaps surprisingly, R^{cl} is not closely related to the equity premium ψ. In the top panel of Figure 3, R^{cl} is independent of χ and thus constant at 10, while ψ varies by a factor of about ten. In the middle and bottom panels, R^{cl} grows linearly along with R^{fl} at a rate much greater than ψ. I discuss the reasons for this divergence between R^{cl} and the equity premium in more detail below, but the important takeaway from Figure 3 and the examples of this section is that the traditional, fixed-labor measure of risk aversion, R^{fl} is unrelated to the risk premium on assets in the model. In contrast, the consumption-wealth measure of relative risk aversion, R^c, which takes into account households' ability to self-insure portfolio fluctuations with changes in labor, tracks the equity premium in the model closely.

4.3 Relative Risk Aversion R^c vs. R^{cl} and the Equity Premium

It may seem surprising that R^{cl} is not more closely related to the equity premium in Figure 3,

[37] Gourio sets $1 - \widetilde{\alpha} = \gamma + \alpha(1-\gamma) = 10$.

given that consumption and leisure form a composite good in those preferences. Instead, the consumption-wealth risk aversion coefficient, R^c, provides the better measure. Looking at the decomposition of the equity premium provided by Corollary 8, what Figure 3 is saying is that the covariance $\mathrm{Cov}_t(dr^i_{t+1}, d\hat{A}_{t+1}/A^c)$ is much closer to being invariant with respect to changes in the household's preference parameters than is the covariance $\mathrm{Cov}_t(dr^i_{t+1}, d\hat{A}_{t+1}/A^{cl})$.[38] In this section, I explore and discuss the reasons for this result.

Note first that—unlike the traditional, fixed-labor measure R^{fl}—both R^c and R^{cl} recognize that households will vary their labor supply to insure themselves from portfolio fluctuations. The issue here is simply whether the value of leisure should be included in household wealth when measuring relative risk aversion, with R^{cl} including the value of leisure and R^c excluding it.

In a model with two consumption goods (and no labor) and period utility $u(c_{1t}, c_{2t}) = \left(c_{1t}^{\chi} c_{2t}^{1-\chi}\right)^{1-\gamma}/(1-\gamma)$, it would seem bizarre to equate household wealth to the present value of consumption of one of the goods, excluding the value of the other. Yet that is essentially what the results in Figure 3 and Example 5 suggest.

The key difference in Example 5 is that consumption and leisure appear separately elsewhere in the model (e.g., in the production function), which is inconsistent with the composite good interpretation. In a model with two consumption goods, varying the parameter χ between 0 and 1 might change the relative sizes of the two consumption good sectors in steady state, but would not have any aggregate general equilibrium implications. In contrast, varying the parameter χ in Example 5 has important general equilibrium effects on steady-state capital, labor, wealth, and other aggregate variables.[39]

To see the effects of χ on the steady state and the covariance term $\mathrm{Cov}_t(dr^i_{t+1}, d\hat{A}_{t+1})$ in Example 5, start by computing the model's steady state. The steady-state interest rate $r = (1-\beta)/\beta$ and marginal product of capital $r^k = (1-\zeta)y/k$, so the output-capital ratio satisfies

$$\frac{y}{k} = \frac{1}{1-\zeta}\left(\frac{1-\beta}{\beta} + \delta\right). \tag{57}$$

From the production function, $(l/k) = (y/k)^{1/\zeta}$, and the aggregate resource constraint implies $(c/k) = (y/k) + \delta$. Thus, the ratios y/k, l/k, and c/k are all invariant with respect to χ, and

[38] As discussed below, the second covariance term in Corollary 8, $\mathrm{Cov}_t(dr^i_{t+1}, d\Phi_{t+1})$, does not vary much with changes in the household's preference parameters in Figure 3.

[39] In partial equilibrium, the interpretation of consumption and leisure as a composite good for the household in Example 5 is valid. The issue is that the composite good interpretation is not valid in the general equilibrium of the model and the graphs in Figure 3 plot the general equilibrium relationship between the equity premium (or risk aversion) and the parameters χ, γ, and α.

so is the steady-state wage $w = \zeta \frac{(y/k)}{(l/k)}$. Finally, the household's period utility function implies $\chi w(1-l) = (1-\chi)c$, and thus

$$k \;=\; \frac{w}{w\,(l/k) + \dfrac{1-\chi}{\chi}(c/k)}\,. \tag{58}$$

The wage w and ratios l/k and c/k are invariant with respect to χ, so the aggregate equilibrium level of k is increasing in χ, ranging from 0 to $(y/k)^{-1/\zeta}$ as χ ranges from 0 to 1.

Thus, varying the parameter χ in Example 5 changes not just the composition of the consumption-leisure aggregate good, but also the equilibrium levels of k and household wealth A^c and A^{cl}, among other variables. This, in turn, changes the crucially important covariance $\mathrm{Cov}_t(dr^i_{t+1}, d\hat{A}_{t+1})$ in Proposition 7. In particular, $\mathrm{Cov}_t(dr^i_{t+1}, d\hat{A}_{t+1})$ is roughly proportional to steady-state k, because $d\hat{A}_{t+1} = da_{t+1} + E_{t+1}\sum_{k=1}^{\infty}(1+r)^{-k}(l\,dw_{t+k} + adr_{t+k})$ scales about linearly with k.[40]

Finally, household wealth A^c is proportional to k.[41] As a result, $\mathrm{Cov}_t(dr^i_{t+1}, d\hat{A}_{t+1}/A^c)$ in Corollary 8 is roughly invariant with respect to χ, implying a tight, linear relationship between $R^c(a;\theta)$ and the equity premium ψ.[42] This close relationship is clearly visible in Figure 3.

By contrast, A^{cl}, the leisure-inclusive measure of household wealth, is *not* proportional to k. The value of leisure, $w(1-l)$, *decreases* with k (because w is invariant and $1-l$ decreases), while nonhuman wealth increases with k. As a result, A^{cl} has no simple relationship to k and $\mathrm{Cov}_t(dr^i_{t+1}, d\hat{A}_{t+1}/A^{cl})$ varies substantially with changes in χ. Thus, there is no stable relationship between R^{cl} and the equity premium in Corollary 8 and Example 5, as is evident in Figure 3.

Intuitively, consumption and leisure do not form a true composite good in the model because labor appears separately in the production function. Thus, a composite-good measure of risk aversion R^{cl} is not necessarily the best measure and in fact does not match the equity premium in Figure 3. Instead, the consumption-wealth coefficient of relative risk aversion, R^c—which recognizes the household's flexible labor margin but excludes the value of leisure from total household wealth—is to be more closely related to the equity premium for these preferences.

[40] Household assets $a = k$ and the ratio l/k is constant, so a and l scale linearly with k. (Labor scales linearly up to its maximum value $l = 1$, which is attained when $\chi = 1$ and $k = 1/(l/k)$.) In contrast, dr_{t+1} and dw_{t+1} hardly change with k because the marginal products of capital and labor, $(1-\zeta)y_t/k_t$ and $\zeta y_t/l_t$, are invariant to changes in steady-state k. The term da_{t+1} grows about linearly with k because technology shocks in the model are multiplicative, so the effects of technology shocks scale. Thus, $d\hat{A}_{t+1}$ scales about linearly with k. The return r^i_{t+1} on the consumption claim hardly changes with k because both sides of the household's Euler equation scale linearly with k. Thus, $\mathrm{Cov}_t(dr^i_{t+1}, d\hat{A}_{t+1})$ varies roughly linearly with k.

[41] Because consumption and hence the present discounted value of consumption scale linearly with k.

[42] The second covariance term in Corollary 8, $\mathrm{Cov}_t(dr^i_{t+1}, d\Phi_{t+1})$, is not strictly invariant to changes in χ, but this term is much smaller than the first and thus does not have a substantial effect on ψ in Figure 3.

Of course, the equity premium depends not just on R^c but also on the two covariance terms in Corollary 8—the covariance of the equity return with household wealth and with changes in purchasing opportunities. To the extent that these covariances change as parameters of any given model are varied, the relationship between the equity premium and R^c will be weaker. However, for standard macroeconomic models like those considered in this section, the risk aversion measure R^c seems to provide a good benchmark.

5. Risk Aversion Away from the Steady State

The closed-form expressions for risk aversion derived in Section 3 hold exactly only at the model's nonstochastic steady state. For values of $(a_t; \theta_t)$ away from steady state, these expressions are only approximations. In this section, I evaluate the accuracy of those approximations by computing risk aversion numerically away from the steady state for the standard real business cycle model described above.

The setup and parameterization of the model are the same as described previously. I assume that households have the same additively separable preferences as in Examples 3–4, with parameter values $\gamma = 5$, $\chi = 1.5$, and $\alpha = -10$. The state variables of the model are k_t and Z_t.[43] The household's consumption-wealth coefficient of relative risk aversion at the steady state, $R^c(k; Z)$, is given by equation (35); for the parameter values above, this implies $R^c(k; Z) = 17.76$, a little more than one-third the traditional measure of $1 - \widetilde{\alpha} = \gamma + \alpha(1 - \gamma) = 45$.

For values of $(k_t; Z_t)$ away from the steady state, equations (9) and (11)–(15) remain valid, and can be used to compute $R^c(k_t; Z_t)$ numerically. I append equations for R^c, V_1, V_{11}, λ_t, and $\partial c_t^* / \partial a_t$ to the standard set of RBC equilibrium conditions and solve them using the same fifth-order perturbation method as in the previous section. (See the Appendix for a complete list of equations and additional details regarding the numerical solution algorithm.)

[Figure 4 about here]

Figure 4 graphs the result as a function of $\log(k_t/k)$ and $\log Z_t$ over a wide range of values for these variables, about ± 10 standard deviations (equal to about ± 38 percent and ± 23 percent

[43] The household's endogenous state variable is its own holdings of capital, k_t. The exogenous state variables of the model are Z_t and the aggregate capital stock, K_t. Thus, the state vector of the household's optimization problem could be written more precisely as $(k_t; Z_t, K_t)$, or even $(k_t; Z_t, K_t, \sigma_\varepsilon^2)$, since the nonstochastic steady state requires setting $\sigma_\varepsilon^2 = 0$. However, in equilibrium, $k_t = K_t$, so for simplicity I write the state vector in this example as $(k_t; Z_t)$.

in logarithmic terms for $\log k_t$ and $\log Z_t$, respectively).[44] The horizontal dashed black lines in Figure 4 report the constant, closed-form value for risk aversion at the nonstochastic steady state, $R^c(k; Z)$, equal to 17.76. The solid red lines in the figure plot the numerical solution for $R^c(k_t; Z_t)$ for general values of k_t and Z_t.[45] The key point of Figure 4 is that, even over the very wide range of values of the state variables considered, the household's coefficient of relative risk aversion ranges between about 17.5 and 18.1, very close to $R^c(k; Z)$, and never near the traditional, fixed-labor value of $R^{fl} = 45$. Thus, the closed-form expressions in Section 3 seem to provide a good approximation to the household's risk aversion in a standard model even far away from steady state.

[Figure 5 about here]

It's also interesting that the household's risk aversion is countercyclical with respect to the state variables k_t and Z_t. This can be seen most clearly in Figure 5, which depicts the household's coefficient of absolute risk aversion, $R^a(k_t; Z_t)$, over the same range of values for k_t and Z_t as in Figure 4. The absolute risk aversion coefficient of .09 implies that the household is willing to pay about 9 cents to avoid a fair gamble with a standard deviation of one dollar. This willingness to pay varies from about 7 to 12 cents over the range of values for the state variables in Figure 5, with higher values of the states corresponding to higher household wealth and lower risk aversion.

Looking back at Figure 4, relative risk aversion is not countercyclical in that figure with respect to k_t because household wealth—and thus the size of the hypothetical gamble faced by the household—is increasing in k_t and Z_t. Indeed, for higher k_t, the increase in wealth is sufficiently large that the household's relative risk aversion increases with k_t, even though absolute risk aversion decreases.

6. Balanced Growth

In previous sections, I abstracted from growth for simplicity, but the results carry through essentially unchanged to the case of balanced growth. I briefly collect the corresponding expressions in this section and provide proofs in the Appendix.

[44] The unconditional standard deviations of $\log Z_t$ and $\log(k_t/k)$ are about 2.3 and 3.8 percent, respectively. The ergodic mean of $\log Z_t$ is zero and that of $\log(k_t/k)$ is about .006, or 0.6 percent.

[45] The red lines do not intersect the black lines at the vertical axis because c_t^* and l_t^* evaluated at $k_t = k$ and $Z_t = Z$ do not equal the nonstochastic steady state values c and l due to the presence of uncertainty (e.g., precautionary savings).

King, Plosser, and Rebelo (1988, 2002) provide a detailed discussion of balanced growth. Along a balanced growth path, $x \in \{l, r\}$ satisfies $x_{t+k} = x_t$ for $k = 1, 2, \ldots$, and I drop the time subscript to denote the constant value. For $x \in \{a, c, w, d\}$, $x_{t+k} = G^k x_t$ for $k = 1, 2, \ldots$, for some $G \in (0, 1+r)$, and I use x_t^{bg} to denote the balanced growth path value. For notational simplicity, I denote the balanced growth path value of θ_t by θ_t^{bg}, although the elements of θ may grow at different constant rates over time (or remain constant).

Lemma 9. *Given Assumptions 1–7 and 8′, for all $k = 1, 2, \ldots$ along the balanced growth path: i) $\lambda_{t+k}^{bg} = G^{-k}\lambda_t^{bg}$, where λ_t^{bg} denotes the balanced growth path value of λ_t, ii) $\partial c_{t+k}^*/\partial a_t = G^k \partial c_t^*/\partial a_t$, iii) $\partial l_{t+k}^*/\partial a_t = \partial l_t^*/\partial a_t$, and iv) $\partial c_t^*/\partial a_t = (1 + r - G)/(1 + w_t^{bg}\lambda_t^{bg})$.*

Proof: See Appendix.

Note that $w_t^{bg}\lambda_t^{bg}$ in Lemma 9 is constant over time because w and λ grow at reciprocal rates. The larger is G, the smaller is $\partial c_t^*/\partial a_t$, since the household chooses to absorb a greater fraction of asset shocks in future periods.

Proposition 10. *Given Assumptions 1–7 and 8′, absolute risk aversion, evaluated along the balanced growth path, satisfies*

$$R^a(a_t^{bg}; \theta_t^{bg}) \; = \; \frac{-V_{11}(a_{t+1}^{bg}; \theta_{t+1}^{bg})}{V_1(a_{t+1}^{bg}; \theta_{t+1}^{bg})} \; + \; \alpha \frac{V_1(a_{t+1}^{bg}; \theta_{t+1}^{bg})}{V(a_{t+1}^{bg}; \theta_{t+1}^{bg})} \tag{59}$$

and

$$R^a(a_t^{bg}; \theta_t^{bg}) \; = \; \frac{-u_{11} + \lambda_t^{bg} u_{12}}{u_1} \, \frac{\frac{1+r}{G} - 1}{1 + w_t^{bg}\lambda_t^{bg}} \; + \; \alpha \left(\frac{1+r}{G} - 1 \right) \frac{u_1}{u} \, , \tag{60}$$

where u_i and u_{ij} denote the corresponding partial derivatives of u evaluated at (c_t^{bg}, l). If $u(c_t, l_t) = \log c_t + v(\bar{l} - l_t)$ for some function v, then u in (60) must be interpreted to mean $\log c_t + v(\bar{l} - l_t) + \frac{\log G}{1 - \frac{G}{1+r}}$.

Proof: See Appendix.

Note that (60) agrees with Proposition 2 when $G = 1$. The larger is G, the smaller is R^a, since larger G implies greater household wealth and ability to absorb shocks to asset values.

Corollary 11. *Given Assumptions 1–7 and 8′, relative risk aversion, evaluated along the balanced growth path, satisfies*

$$R^c(a_t^{bg}; \theta_t^{bg}) \; = \; \frac{-u_{11} + \lambda_t^{bg} u_{12}}{u_1} \, \frac{c_t^{bg}}{1 + w_t^{bg}\lambda_t^{bg}} \; + \; \alpha \frac{c_t^{bg} u_1}{u} \tag{61}$$

and

$$R^{cl}(a_t^{bg}; \theta_t^{bg}) \; = \; \frac{-u_{11} + \lambda_t^{bg} u_{12}}{u_1} \, \frac{c_t^{bg} + w_t^{bg}(\bar{l} - l)}{1 + w_t^{bg}\lambda_t^{bg}} \; + \; \alpha \frac{\big(c_t^{bg} + w_t^{bg}(\bar{l} - l)\big) u_1}{u} \, . \tag{62}$$

If $u(c_t, l_t) = \log c_t + v(\bar{l} - l_t)$ *for some function* v*, then* u *in* (61)–(62) *must be interpreted to mean* $\log c_t + v(\bar{l} - l_t) + \frac{\log G}{1 - \frac{G}{1+r}}$.

PROOF: See Appendix.

Thus, the expressions for relative risk aversion are essentially unchanged by balanced growth.

7. Multiplier Preferences

Multiplier preferences are a version of generalized recursive preferences defined by Hansen and Sargent (2001) and Strzalecki (2011). I briefly review those preferences here and derive the corresponding expressions for risk aversion with labor.

Households with multiplier preferences order state-contingent consumption and labor plans according to the recursive functional

$$\widetilde{W}(c^t, l^t) \;=\; (1-\beta)\, u(c_t, l_t) \;-\; \beta\phi^{-1} \log E_t \exp\big(-\phi \widetilde{W}(c^{t+1}, l^{t+1})\big), \tag{63}$$

rather than (3), where β is the household's discount factor and $\phi \in \mathbb{R}$. The preferences (63) can be regarded as a special case of (5), corresponding to $\rho = 0$. Let $W(a_t; \theta_t)$ denote the maximized value of (63), subject to (1)–(2):

$$W(a_t; \theta_t) \;=\; \max_{(c_t, l_t) \in \Gamma(a_t; \theta_t)} \; (1-\beta)\, u(c_t, l_t) \;-\; \beta\phi^{-1} \log E_t \exp\big(-\phi W(a_{t+1}; \theta_{t+1})\big). \tag{64}$$

Hansen and Sargent (2001) show how (63)–(64) can be derived from microfoundations based on household optimization in the presence of concerns regarding model misspecification.[46] Maximizing (64) instead of expected utility ensures that the household achieves a reasonable discounted sum of utility flows for a range of empirically plausible processes for θ_t.

As ϕ approaches 0, (64) converges to expected utility. For $\phi \neq 0$, the intertemporal elasticity of substitution is the same as for expected utility, but the household's risk aversion can be amplified (or attenuated) by the additional curvature parameter ϕ.

From a practical perspective, an advantage of multiplier preferences is that they are well-defined even when u takes on both positive and negative values, so Assumption 2 can be dropped. Modifying the other assumptions and definitions to correspond to W rather than V gives the following:

[46] These microfoundations can be used to derive values of $\phi \geq 0$. The case $\phi < 0$, corresponding to risk-loving behavior, cannot be microfounded this way.

Proposition 12. *Let $(a_t;\theta_t)$ be an interior point of X. Given Assumptions 1 and 3–6, $\hat{W}(a_t;\theta_t;\sigma)$, $\mu(a_t;\theta_t;\sigma)$, and $R^a(a_t;\theta_t)$ exist, and*

$$R^a(a_t;\theta_t) \;=\; \frac{-E_t \,\exp\big(-\,\phi W(a^*_{t+1};\theta_{t+1})\big)\,\big[W_{11}(a^*_{t+1};\theta_{t+1}) \,-\, \phi W_1(a^*_{t+1};\theta_{t+1})^2\big]}{E_t\,\exp\big(-\,\phi W(a^*_{t+1};\theta_{t+1})\big)W_1(a^*_{t+1};\theta_{t+1})}\,. \tag{65}$$

Given Assumptions 7–8, (65) *can be evaluated at the steady state to yield:*

$$R^a(a;\theta) \;=\; \frac{-W_{11}(a;\theta)}{W_1(a;\theta)} \,+\, \phi W_1(a;\theta)\,. \tag{66}$$

Proof: The proof follows along exactly the same lines as Proposition 1.

Even though the preferences (64) can be derived from a concern for robustness rather than risk, the household acts in a way that is observationally equivalent to having higher risk aversion. That is, if one confronts a Hansen-Sargent household with the hypothetical gamble in (7), the household's concerns about the stochastic process $\{\theta_t\}$ manifest themselves as an increased aversion to the gamble; as a result, the household behaves exactly as if it were certain about the economic environment but had a higher level of risk aversion governed by ϕ. Higher values of ϕ correspond to higher levels of risk aversion, with sufficiently negative values of ϕ corresponding to risk-loving behavior.

Proposition 13. *Given Assumptions 1 and 3–8, the household's coefficient of absolute risk aversion for multiplier preferences, evaluated at steady state, satisfies*

$$R^a(a;\theta) \;=\; \frac{-u_{11}+\lambda u_{12}}{u_1}\,\frac{r}{1+w\lambda} \,+\, \phi\, r u_1. \tag{67}$$

Proof: The proof follows along the same lines as Proposition 2.

Corollary 14. *Given Assumptions 1 and 3–8, the household's relative risk aversion for multiplier preferences, evaluated at steady state, satisfies*

$$R^c(a;\theta) \;=\; \frac{-u_{11}+\lambda u_{12}}{u_1}\,\frac{c}{1+w\lambda} \,+\, \phi\, c u_1 \tag{68}$$

and

$$R^{cl}(a;\theta) \;=\; \frac{-u_{11}+\lambda u_{12}}{u_1}\,\frac{c+w(\bar{l}-l)}{1+w\lambda} \,+\, \phi\big(c+w(\bar{l}-l)\big)u_1. \tag{69}$$

A feature of multiplier preferences worth emphasizing is that additive shifts of the period utility function u have no effect on risk aversion, while multiplicative scalings of u *do* affect risk aversion. (For standard Epstein-Zin-Weil preferences, it is the other way around: multiplicative transformations of u have no effect, while additive shifts affect risk aversion.) The important point to note is that (67)–(69) *only* hold when the period utility function $u(c_t, l_t)$ is premultiplied by $(1-\beta)$, as in (63) and (64). Without that scaling factor, the second terms of (67)–(69) each need to be multiplied by $(1-\beta)^{-1} = \dfrac{1+r}{r}$. If $\beta = .99$, this is observationally equivalent to increasing ϕ by a factor of 100, a huge increase in risk aversion for what might seem like a simple renormalization. Note that some authors (e.g., Tallarini, 2001; Barillas, Hansen, and Sargent, 2009) pre-multiply their period utility functions by a factor of $(1-\beta)$, while others (e.g., Boyarchenko, 2012; Bidder and Smith, 2013) do not, so there is no convention in the literature, and the discussion in these papers typically gives the reader no indication that risk aversion differs by a factor of about 100 across the two alternatives. Thus, it's important that researchers bear in mind the dramatic effect different scalings of u have on risk aversion and risk premia in these models.

Example 6. Tallarini (2000) considers the multiplier specification (67) with period utility

$$u(c_t, l_t) \;=\; \frac{1}{1+\xi}\log c_t + \frac{\xi}{1+\xi}\log(\bar{l} - l_t), \tag{70}$$

where $\xi \geq 0$. The household's consumption-wealth coefficient of relative risk aversion is given by

$$R^c(a;\theta) \;=\; \frac{-u_{11} + \lambda u_{12}}{u_1}\,\frac{c}{1+w\lambda} \;+\; \phi\, c u_1 \;=\; \frac{1+\phi}{1+\xi}\,, \tag{71}$$

while including the value of leisure in household wealth gives

$$R^{cl}(a;\theta) \;=\; \frac{-u_{11} + \lambda u_{12}}{u_1}\,\frac{c + w(\bar{l} - l)}{1+w\lambda} \;+\; \phi\big(c + w(\bar{l} - l)\big)u_1 \;=\; 1+\phi\,. \tag{72}$$

Neither of these equals the traditional, fixed-labor measure of risk aversion reported by Tallarini,

$$R^{fl}(a;\theta) \;=\; \frac{-cu_{11}}{u_1} \;+\; \phi\, c u_1 \;=\; 1 + \frac{\phi}{1+\xi}\,. \tag{73}$$

This last measure ignores the fact that households will vary their labor endogenously in response to shocks. Note that $R^c \leq R^{fl}$, as always, although in this particular example the difference is not very large quantitatively.

8. Discussion and Conclusions

There are several points to take away from the analysis above. First, traditional studies of risk aversion, such as Arrow (1965), Pratt (1964), Epstein and Zin (1989), and Weil (1989), assume that household labor supply is fixed, and thus ignore households' ability in standard macroeconomic models to partially offset shocks to asset values by varying their labor. As a result, traditional fixed-labor measures of risk aversion are not representative of households' aversion to holding risky assets when labor supply can vary. For reasonable parameterizations, these traditional measures of risk aversion can overstate the household's actual aversion to monetary gambles by a factor of as much as ten, as in Figure 3. Fixed-labor measures of risk aversion are also unrelated to the equity premium in a standard RBC model, while the flexible-labor measure R^c derived above is much more closely related.

Second, applying the Epstein-Zin-Weil fixed-labor measure of risk aversion to a Cobb-Douglas aggregate of consumption and leisure, as is sometimes done in the literature, is also problematic. If labor and consumption appear separately elsewhere in the model, such as in the production function, then consumption and leisure do not form a true composite good in the model. As a result, a composite-good measure of risk aversion is not necessarily appropriate, and in fact, turns out to be poorly correlated with the equity premium. In contrast, the consumption-wealth coefficient of relative risk aversion R^c defined in the present paper is more closely related. This measure recognizes the household's ability to partially offset portfolio shocks by varying labor supply, but—unlike the Cobb-Douglas aggregate—excludes the value of leisure from household wealth.

Third, for multiplier preferences, including or excluding a scale factor of $1 - \beta$ in period utility can lead to huge differences in risk aversion, by a factor of about 100. Thus, researchers must be very careful to account correctly for any scale factor in utility when computing risk aversion in models with multiplier preferences.

Fourth, the flexible-labor risk aversion measure R^c is less than both the traditional, fixed-labor measure, R^{fl}, and the Cobb-Douglas aggregate measure, R^{cl}, described above. As a result, many studies in the macroeconomics, macro-finance, and international finance literatures may be significantly overstating the relevant degree of risk aversion in their models.

Finally, the closed-form expressions for risk aversion I derive above, and the methods of the paper more generally, are potentially useful for asset pricing in any dynamic model with multiple goods in the utility function. Models with home production, money in the utility function, or tradeable and nontradeable goods can imply very different household attitudes toward risk than traditional measures of risk aversion might suggest.

Appendix: Proofs of Propositions and Numerical Solution Details

Proof of Proposition 1

Since $(a_t;\theta_t)$ is an interior point of X, $V(a_t+\frac{\sigma\underline{\varepsilon}}{1+r_t};\theta_t)$ and $V(a_t+\frac{\sigma\overline{\varepsilon}}{1+r_t};\theta_t)$ exist for sufficiently small σ, and $V(a_t+\frac{\sigma\underline{\varepsilon}}{1+r_t};\theta_t) \le \hat{V}(a_t;\theta_t;\sigma) \le V(a_t+\frac{\sigma\overline{\varepsilon}}{1+r_t};\theta_t)$, hence $\hat{V}(a_t;\theta_t;\sigma)$ exists. Moreover, since $V(\cdot;\cdot)$ is continuous and increasing in its first argument, the intermediate value theorem implies there exists a unique $-\mu(\sigma) \in [\sigma\underline{\varepsilon}, \sigma\overline{\varepsilon}]$ with $V(a_t-\frac{\mu(\sigma)}{1+r_t};\theta_t) = \hat{V}(a_t;\theta_t;\sigma)$.

For generalized recursive preferences, the household's first-order optimality conditions for c_t^* and l_t^*,

$$u_1(c_t^*,l_t^*) \;=\; \beta\big(E_tV(a_{t+1}^*;\theta_{t+1})^{1-\alpha}\big)^{\alpha/(1-\alpha)}E_tV(a_{t+1}^*;\theta_{t+1})^{-\alpha}V_1(a_{t+1}^*;\theta_{t+1}), \tag{A1}$$

$$u_2(c_t^*,l_t^*) \;=\; -\beta w_t\big(E_tV(a_{t+1}^*;\theta_{t+1})^{1-\alpha}\big)^{\alpha/(1-\alpha)}E_tV(a_{t+1}^*;\theta_{t+1})^{-\alpha}V_1(a_{t+1}^*;\theta_{t+1}), \tag{A2}$$

are slightly more complicated than the case of expected utility considered in Swanson (2012). Note that (A1) and (A2) are related by the usual $u_2(c_t^*,l_t^*) = -w_t u_1(c_t^*;l_t^*)$, and when $\alpha = 0$, (A1) and (A2) reduce to the standard optimality conditions for expected utility.

For an infinitesimal fee $d\mu$ in (8), the first-order change in household welfare (4) is given by

$$-V_1(a_t;\theta_t)\,\frac{d\mu}{1+r_t}. \tag{A3}$$

Differentiating (6) with respect to a_t yields

$$\begin{aligned} V_1(a_t;\theta_t) \;=\;& u_1(c_t^*,l_t^*)\frac{\partial c_t^*}{\partial a_t} \;+\; u_2(c_t^*,l_t^*)\frac{\partial l_t^*}{\partial a_t} \\ &+\beta\big(E_tV(a_{t+1}^*;\theta_{t+1})^{1-\alpha}\big)^{\alpha/(1-\alpha)}E_tV(a_{t+1}^*;\theta_{t+1})^{-\alpha}V_1(a_{t+1}^*;\theta_{t+1})\left[(1+r_t)-\frac{\partial c_t^*}{\partial a_t}+w_t\frac{\partial l_t^*}{\partial a_t}\right]. \end{aligned} \tag{A4}$$

Applying (A1)–(A2) to (A4) gives the envelope theorem,

$$V_1(a_t;\theta_t) \;=\; \beta(1+r_t)\big(E_tV(a_{t+1}^*;\theta_{t+1})^{1-\alpha}\big)^{\alpha/(1-\alpha)}E_tV(a_{t+1}^*;\theta_{t+1})^{-\alpha}V_1(a_{t+1}^*;\theta_{t+1}) \tag{A5}$$

and the Benveniste-Scheinkman equation (11),

$$V_1(a_t;\theta_t) \;=\; (1+r_t)u_1(c_t^*,l_t^*)\,. \tag{A6}$$

From (A5), (A3) equals

$$-\beta\big(E_tV(a_{t+1}^*;\theta_{t+1})^{1-\alpha}\big)^{\alpha/(1-\alpha)}E_tV(a_{t+1}^*;\theta_{t+1})^{-\alpha}V_1(a_{t+1}^*;\theta_{t+1})\,d\mu\,. \tag{A7}$$

Turning now to the gamble in (7), the household's optimal choices for consumption and labor in period t, c_t^* and l_t^*, will generally depend on the size of the gamble σ—for example, the household may undertake precautionary saving when faced with this gamble. Thus, in this section we write $c_t^* \equiv c^*(a_t;\theta_t;\sigma)$ and $l_t^* \equiv l^*(a_t;\theta_t;\sigma)$ to emphasize this dependence on σ. The household's value function, inclusive of the one-shot gamble in (7), satisfies

$$\hat{V}(a_t;\theta_t;\sigma) = u(c_t^*,l_t^*) + \beta E_tV(a_{t+1}^*;\theta_{t+1}), \tag{A8}$$

where $a_{t+1}^* \equiv (1+r_t)a_t + w_tl_t^* + d_t - c_t^*$. Because (7) describes a one-shot gamble in period t, it affects assets a_{t+1}^* in period $t+1$ but otherwise does not affect the household's optimization problem from period $t+1$ onward; as a result, the household's value-to-go at time $t+1$ is just $V(a_{t+1}^*;\theta_{t+1})$, which does not depend on σ except through a_{t+1}^*.

Differentiating (A8) with respect to σ, the first-order effect of the gamble on household welfare is:

$$\left[u_1\frac{\partial c^*}{\partial\sigma} + u_2\frac{\partial l^*}{\partial\sigma} + \beta\big(E_tV^{1-\alpha}\big)^{\alpha/(1-\alpha)}E_tV^{-\alpha}V_1\cdot\big(w_t\frac{\partial l^*}{\partial\sigma} - \frac{\partial c^*}{\partial\sigma} + \varepsilon_{t+1}\big)\right]d\sigma, \tag{A9}$$

where the arguments of u_1, u_2, V, and V_1 are suppressed to simplify notation. Optimality of c_t^* and l_t^* implies that the terms involving $\partial c^*/\partial\sigma$ and $\partial l^*/\partial\sigma$ cancel, as in the usual envelope theorem (these derivatives vanish at $\sigma = 0$ anyway, for the reasons discussed below). Moreover, $E_t V^{-\alpha} V_1 \varepsilon_{t+1} = 0$ because ε_{t+1} is independent of θ_{t+1} and a_{t+1}^*, evaluating the latter at $\sigma = 0$. Thus, the first-order cost of the gamble is zero, as in Arrow (1965) and Pratt (1964).

To second order, the effect of the gamble on household welfare is

$$\begin{aligned}\Bigg\{ & u_{11}\left(\frac{\partial c^*}{\partial\sigma}\right)^2 + 2u_{12}\frac{\partial c^*}{\partial\sigma}\frac{\partial l^*}{\partial\sigma} + u_{22}\left(\frac{\partial l^*}{\partial\sigma}\right)^2 + u_1\frac{\partial^2 c^*}{\partial\sigma^2} + u_2\frac{\partial^2 l^*}{\partial\sigma^2} \\ & + \alpha\beta(E_t V^{1-\alpha})^{(2\alpha-1)/(1-\alpha)}\left[E_t V^{-\alpha} V_1 \cdot \left(w_t\frac{\partial l^*}{\partial\sigma} - \frac{\partial c^*}{\partial\sigma} + \varepsilon_{t+1}\right)\right]^2 \\ & - \alpha\beta(E_t V^{1-\alpha})^{\alpha/(1-\alpha)} E_t V^{-\alpha-1}\left[V_1 \cdot \left(w_t\frac{\partial l^*}{\partial\sigma} - \frac{\partial c^*}{\partial\sigma} + \varepsilon_{t+1}\right)\right]^2 \\ & + \beta(E_t V^{1-\alpha})^{\alpha/(1-\alpha)} E_t V^{-\alpha} V_{11} \cdot \left(w_t\frac{\partial l^*}{\partial\sigma} - \frac{\partial c^*}{\partial\sigma} + \varepsilon_{t+1}\right)^2 \\ & + \beta(E_t V^{1-\alpha})^{\alpha/(1-\alpha)} E_t V^{-\alpha} V_1 \cdot \left(w_t\frac{\partial^2 l^*}{\partial\sigma^2} - \frac{\partial^2 c^*}{\partial\sigma^2}\right)\Bigg\}\frac{d\sigma^2}{2}. \end{aligned} \tag{A10}$$

The terms involving $\partial^2 c^*/\partial\sigma^2$ and $\partial^2 l^*/\partial\sigma^2$ cancel due to the optimality of c_t^* and l_t^*. The derivatives $\partial c^*/\partial\sigma$ and $\partial l^*/\partial\sigma$ vanish at $\sigma = 0$ (there are two ways to see this: first, the linearized version of the model is certainty equivalent; alternatively, if the distribution of ε is symmetric about zero, the gamble in (7) is isomorphic for positive and negative σ, hence c^* and l^* must be symmetric about $\sigma = 0$, implying the derivatives vanish). Finally, ε_{t+1} is independent of θ_{t+1} and a_{t+1}^*, evaluating the latter at $\sigma = 0$. Since ε_{t+1} has unit variance, (A10) reduces to

$$\beta(E_t V^{1-\alpha})^{\alpha/(1-\alpha)}\left(E_t V^{-\alpha} V_{11} - \alpha E_t V^{-\alpha-1} V_1^2\right)\frac{d\sigma^2}{2}. \tag{A11}$$

Equating (A7) to (A11) allows us to solve for $d\mu$ as a function of $d\sigma^2$. Thus, $\lim_{\sigma\to 0} 2\mu(\sigma)/\sigma^2$ exists and is given by

$$\frac{-E_t V^{-\alpha} V_{11} + \alpha E_t V^{-\alpha-1} V_1^2}{E_t V^{-\alpha} V_1}. \tag{A12}$$

Since (A12) is already evaluated at $\sigma = 0$, to evaluate it at the nonstochastic steady state, set $a_{t+1} = a$ and $\theta_{t+1} = \theta$ to get

$$\frac{-V_{11}(a;\theta)}{V_1(a;\theta)} + \alpha\frac{V_1(a;\theta)}{V(a;\theta)}. \tag{A13}$$

Proof of Lemma 2

Equations (A1), (A4), and the envelope theorem imply the household's intertemporal optimality (Euler) condition,

$$u_1(c_t^*, l_t^*) \;=\; \beta\big(E_t V(a_{t+1}^*;\theta_{t+1})^{1-\alpha}\big)^{\alpha/(1-\alpha)} E_t V(a_{t+1}^*;\theta_{t+1})^{-\alpha}(1 + r_{t+1})\, u_1(c_{t+1}^*, l_{t+1}^*). \tag{A14}$$

Differentiating (A14) with respect to a_t at the nonstochastic steady state implies

$$u_{11}\left(\frac{\partial c_t^*}{\partial a_t} - E_t\frac{\partial c_{t+1}^*}{\partial a_t}\right) \;=\; -u_{12}\left(\frac{\partial l_t^*}{\partial a_t} - E_t\frac{\partial l_{t+1}^*}{\partial a_t}\right) \tag{A15}$$

in a neighborhood of the steady state, where the arguments of the u_{ij} are suppressed to reduce notation. Using (14), this implies

$$(u_{11} - \lambda u_{12})\left(\frac{\partial c_t^*}{\partial a_t} - E_t\frac{\partial c_{t+1}^*}{\partial a_t}\right) \;=\; 0 \tag{A16}$$

and thus

$$E_t \frac{\partial c^*_{t+1}}{\partial a_t} \;=\; \frac{\partial c^*_t}{\partial a_t}. \tag{A17}$$

Equations (A14)–(A17) can be iterated forward to yield

$$E_t \frac{\partial c^*_{t+k}}{\partial a_t} \;=\; \frac{\partial c^*_t}{\partial a_t}, \qquad k = 1, 2, \ldots, \tag{A18}$$

whatever the initial response $\partial c^*_t / \partial a_t$. From (14) and (A18), it also follows that

$$E_t \frac{\partial l^*_{t+k}}{\partial a_t} \;=\; \frac{\partial l^*_t}{\partial a_t}, \qquad k = 1, 2, \ldots \tag{A19}$$

It remains to solve for $\partial c^*_t / \partial a_t$. The household's intertemporal budget constraint, evaluated at steady state, implies

$$\frac{1+r}{r} \frac{\partial c^*_t}{\partial a_t} \;=\; (1+r) \;+\; w \frac{1+r}{r} \frac{\partial l^*_t}{\partial a_t}\,. \tag{A20}$$

Substituting (14) into (A20) and solving for $\partial c^*_t / \partial a_t$ yields

$$\frac{\partial c^*_t}{\partial a_t} \;=\; \frac{r}{1+w\lambda}\,. \tag{A21}$$

Proof of Lemma 5

Differentiating the household's Euler equation (A14) at the nonstochastic steady state implies

$$u_{11}(dc^*_t - E_t dc^*_{t+1}) + u_{12}(dl^*_t - E_t dl^*_{t+1}) = \beta u_1 E_t dr_{t+1}, \tag{A22}$$

which, applying (46), becomes

$$(u_{11} - \lambda u_{12})(dc^*_t - E_t dc^*_{t+1}) - \frac{u_1 u_{12}}{u_{22} + w u_{12}}(dw_t - E_t dw_{t+1}) = \beta u_1 E_t dr_{t+1}. \tag{A23}$$

Note that (A23) implies, for each $k = 1, 2, \ldots$,

$$E_t dc^*_{t+k} \;=\; dc^*_t - \frac{u_1 u_{12}}{u_{11} u_{22} - u_{12}^2}(dw_t - E_t dw_{t+k}) - \frac{\beta u_1}{u_{11} - \lambda u_{12}} E_t \sum_{i=1}^{k} dr_{t+i}. \tag{A24}$$

Combining (1)–(2), differentiating, and evaluating at the nonstochastic steady state yields

$$E_t \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} (dc^*_{t+k} - w dl^*_{t+k} - l dw_{t+k} - dd_{t+k} - a dr_{t+k}) \;=\; (1+r)\, da_t. \tag{A25}$$

Substituting (46) and (A24) into (A25), and solving for dc^*_t, yields

$$\begin{aligned} dc^*_t \;=\; & \frac{r}{1+r} \frac{1}{1+w\lambda} \left[(1+r) da_t + E_t \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} \left(l\, dw_{t+k} + dd_{t+k} + a dr_{t+k} \right) \right] \\ & + \frac{u_1 u_{12}}{u_{11} u_{22} - u_{12}^2} dw_t + \frac{1}{1+r} \frac{-u_1}{u_{11} - \lambda u_{12}} E_t \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} \left[\frac{r\lambda}{1+w\lambda} dw_{t+k} - \beta dr_{t+k+1} \right]. \end{aligned} \tag{A26}$$

Proof of Lemma 6

Differentiating equation (6) and evaluating at the nonstochastic steady state implies

$$dV_t \;=\; u_1 dc^*_t + u_2 dl^*_t + \beta E_t dV_{t+1}. \tag{A27}$$

Solving (A27) forward and applying (46) yields

$$dV_t \;=\; \sum_{k=0}^{\infty} \beta^k u_1(1+w\lambda) E_t dc^*_{t+k} \;-\; \sum_{k=0}^{\infty} \beta^k \frac{u_1 u_2}{u_{22}+w u_{12}} E_t dw_{t+k} \,. \tag{A28}$$

Substituting (A24) into (A28) and simplifying yields

$$dV_t \;=\; \frac{1+r}{r} u_1(1+w\lambda) dc^*_t \;-\; \frac{1+r}{r} \frac{u_1^2 u_{12}(1+w\lambda)}{u_{11}u_{22}-u_{12}^2} dw_t + \sum_{k=0}^{\infty} \beta^k \frac{u_1(u_1u_{12}-u_2u_{11})}{u_{11}u_{22}-u_{12}^2} E_t dw_{t+k} \;-\; \frac{u_1^2(1+w\lambda)}{u_{11}-\lambda u_{12}} \frac{1}{1-\beta} \sum_{k=1}^{\infty} \beta^{k+1} E_t dr_{t+k} \,. \tag{A29}$$

Substituting (A26) into (A29) and simplifying gives

$$dV_t \;=\; u_1(1+r) da_t \;+\; u_1 E_t \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} (l dw_{t+k} + dd_{t+k} + a dr_{t+k}). \tag{A30}$$

Proof of Lemma 9

i) The household's Euler equation (A14), evaluated along the (nonstochastic) balanced growth path, implies

$$u_1(c^{bg}_t, l) \;=\; \beta(1+r) u_1(c^{bg}_{t+1}, l) \;=\; \beta(1+r) u_1(G c^{bg}_t, l). \tag{A31}$$

Similarly, for labor,

$$u_2(c^{bg}_t, l) \;=\; \beta(1+r) \frac{w^{bg}_t}{w^{bg}_{t+1}} u_2(c^{bg}_{t+1}, l) \;=\; \beta(1+r) G^{-1} u_2(G c^{bg}_t, l). \tag{A32}$$

As in King, Plosser, and Rebelo (2002), assume that preferences u are consistent with balanced growth for all initial asset stocks and wages in a neighborhood of a^{bg}_t and w^{bg}_t, and hence for all initial values of (c_t, l_t) in a neighborhood of (c^{bg}_t, l). Thus, (A31) and (A32) can be differentiated to yield:

$$u_{11}(c^{bg}_t, l) \;=\; \beta(1+r) G\, u_{11}(G c^{bg}_t, l), \tag{A33}$$

$$u_{12}(c^{bg}_t, l) \;=\; \beta(1+r)\, u_{12}(G c^{bg}_t, l), \tag{A34}$$

$$u_{22}(c^{bg}_t, l) \;=\; \beta(1+r) G^{-1}\, u_{22}(G c^{bg}_t, l). \tag{A35}$$

Substituting (A33)–(A35) into (15) gives

$$\lambda^{bg}_{t+1} \;=\; \frac{w^{bg}_{t+1} u_{11}(c^{bg}_{t+1}, l) + u_{12}(c^{bg}_{t+1}, l)}{u_{22}(c^{bg}_{t+1}, l) + w^{bg}_{t+1} u_{12}(c^{bg}_{t+1}, l)} \;=\; G^{-1} \lambda^{bg}_t \,, \tag{A36}$$

ii) Assumptions 1–6 imply (11)–(15) in the text and the Euler equation (A14). Hence

$$\left(u_{11}(c^{bg}_t, l) - \lambda^{bg}_t u_{12}(c^{bg}_t, l)\right) \frac{\partial c^*_t}{\partial a_t} \;=\; \beta(1+r) \left(u_{11}(c^{bg}_{t+1}, l) - \lambda^{bg}_{t+1} u_{12}(c^{bg}_{t+1}, l)\right) \frac{\partial c^*_{t+1}}{\partial a_t} \,. \tag{A37}$$

Solving for $\partial c^*_{t+1}/\partial a_t$ and using (A33)–(A36) yields $\partial c^*_{t+1}/\partial a_t = G\, \partial c^*_t/\partial a_t$.

iii) Follows from (14), (A33)–(A36), and ii).

iv) Use the household's budget constraint (1)–(2) and ii) to solve for $\partial c^*_t/\partial a_t$.

Proof of Proposition 10

Proposition 1 implies (62). Assumptions 1–6 imply (11)–(15). Substituting (11)–(14) and Lemma 9(iv) into (62) gives

$$R^a(a^{bg}_t; \theta^{bg}_t) \;=\; \frac{-u_{11}(c^{bg}_{t+1}, l) + \lambda^{bg}_{t+1} u_{12}(c^{bg}_{t+1}, l)}{u_1(c^{bg}_{t+1}, l)} \frac{1+r-G}{1+w^{bg}_{t+1}\lambda^{bg}_{t+1}} \;+\; \alpha \frac{(1+r) u_1(c^{bg}_{t+1}, l)}{V(a^{bg}_{t+1}; \theta^{bg}_{t+1})} \,. \tag{A38}$$

Expressing $V(a_{t+1}^{bg};\theta_{t+1}^{bg})$ in terms of period utility u is made slightly more complicated by the presence of balanced growth, since now the arguments of u are not constant but rather grow over time.

King, Plosser, and Rebelo (1988, 2002) show that, to be consistent with balanced growth, $u(c_t, l_t)$ must have the functional form

$$u(c_t, l_t) \;=\; \frac{c_t^{1-\gamma}}{1-\gamma}\, v(\bar{l} - l_t) \tag{A39}$$

or, as $\gamma \to 1$,

$$u(c_t, l_t) \;=\; \log c_t + v(\bar{l} - l_t), \tag{A40}$$

where $v(\cdot)$ in (A39) or (A40) is differentiable, increasing, and concave, but otherwise unrestricted. Since the balanced growth path is nonstochastic, the allowable functional forms for $u(c_t, l_t)$ are the same for the case of generalized recursive preferences as they are for expected utility.

If u has the form (A39), then

$$V(a_t^{bg};\theta_t^{bg}) \;=\; \frac{1}{1-\beta G^{1-\gamma}}\, u(c_t^{bg}, l) \tag{A41}$$

and

$$\beta V(a_{t+1}^{bg};\theta_{t+1}^{bg}) \;=\; V(a_t^{bg};\theta_t^{bg}) - u(c_t^{bg}, l) \;=\; \frac{\beta G^{1-\gamma}}{1-\beta G^{1-\gamma}}\, u(c_t^{bg}, l). \tag{A42}$$

Moreover, $\beta(1+r) = G^{\gamma}$. Substituting (A31), (A33)–(A35), and (A42) into (A38) then completes the proof.

If u has the form (A40), then

$$V(a_t^{bg};\theta_t^{bg}) \;=\; \frac{1}{1-\beta}\, u(c_t^{bg}, l) \;+\; \frac{\beta}{(1-\beta)^2}\, \log G, \tag{A43}$$

$$\beta V(a_{t+1}^{bg};\theta_{t+1}^{bg}) \;=\; \frac{\beta}{1-\beta}\, u(c_t^{bg}, l) \;+\; \frac{\beta}{(1-\beta)^2}\, \log G, \tag{A44}$$

and $\beta(1+r) = G$. Substituting (A31), (A33)–(A36), and (A44) into (A38) yields

$$R^a(a_t^{bg};\theta_t^{bg}) \;=\; \frac{-u_{11} + \lambda_t^{bg} u_{12}}{u_1}\, \frac{\frac{1+r}{G} - 1}{1 + w_t^{bg}\lambda_t^{bg}} \;+\; \alpha\left(\frac{1+r}{G} - 1\right) \frac{u_1}{u + \frac{1+r}{1+r-G}\log G}\,. \tag{A45}$$

This differs from (63) by the addition of the constant term $\frac{\log G}{1-\frac{G}{1+r}}$ to u. Thus, in the case of log preferences, u in (63) must be interpreted to include the additive constant $\frac{\log G}{1-\frac{G}{1+r}}$.

Proof of Corollary 11

As in Definitions 2–3, define wealth A_t^{bg} in beginning- rather than end-of-period-t units; this requires multiplying by $(1+r)^{-1}G$ rather than just $(1+r)^{-1}$. Then the present discounted value of consumption along the balanced growth path is given by $A_t^{bg} = c_t^{bg}/(\frac{1+r}{G} - 1)$, and the present discounted value of leisure by $w_t^{bg}(\bar{l} - l)/(\frac{1+r}{G} - 1)$. Substituting these into Proposition 10 completes the proof.

Numerical Solution Procedure for Sections 4–5

The equations of the RBC model itself are standard:

$$Y_t = Z_t K_{t-1}^{1-\theta} L_t^{\theta}, \tag{A46}$$

$$K_t = (1-\delta)K_{t-1} + Y_t - C_t, \tag{A47}$$

$$\eta L_t^{\chi} / C_t^{-\gamma} = w_t, \tag{A48}$$

$$r_t = (1-\theta)Y_t/K_{t-1} - \delta, \tag{A49}$$

$$w_t = \theta Y_t / L_t, \tag{A50}$$

$$\log Z_t = \rho \log Z_{t-1} + \varepsilon_t, \tag{A51}$$

where, for concreteness, I use the additively separable preference specification from Examples 3 and 5 in (A49) and throughout this section. In equations (A46)–(A51), note that K_{t-1} denotes the capital stock at the beginning of period t (or the end of period $t-1$), so the notation differs slightly from the main text for compatibility with the numerical algorithm below.

Because of the generalized recursive structure of household preferences, the household's Euler equation (A14) involves the value function. Following Rudebusch and Swanson (2012), I use two equations to compute the value function in the model, as follows:

$$V_t \;=\; \frac{C_t^{1-\gamma}}{1-\gamma} - \eta \frac{L_t^{1+\chi}}{1+\chi} + \beta \, \mathrm{VTWIST}_t^{1/(1-\alpha)}, \tag{A52}$$

$$\mathrm{VTWIST}_t \;=\; E_t V_{t+1}^{1-\alpha}. \tag{A53}$$

The household's Euler equation (A14) then can be written as

$$C_t^{-\gamma} \;=\; \beta E_t (1+r_{t+1}) \left(V_{t+1}/\mathrm{VTWIST}_t^{1/(1-\alpha)}\right)^{-\alpha} C_{t+1}^{-\gamma}. \tag{A54}$$

To compute risk aversion in the model, we can append the following auxiliary variables and equations to the system (A46)–(A54):[47]

$$\lambda_t = (\gamma/\chi) L_t / C_t, \tag{A55}$$

$$\mathrm{CARA}_t \;=\; \frac{E_t V_{t+1}^{-\alpha} \left[(1+r_{t+1})(\gamma C_{t+1}^{-\gamma-1} \mathrm{DCDA}_{t+1}) \;+\; \alpha (1+r_{t+1})^2 C_{t+1}^{-2\gamma}/V_{t+1}\right]}{\mathrm{V1EXP}_t}, \tag{A56}$$

$$\mathrm{V1EXP}_t \;=\; E_t V_{t+1}^{-\alpha} (1+r_{t+1}) C_{t+1}^{-\gamma}. \tag{A57}$$

$$\begin{aligned} -\gamma C_t^{-\gamma-1}\, \mathrm{DCDA}_t \;=\; \beta E_t \big[& -\gamma \mathrm{VTWIST}_t^{\alpha/(1-\alpha)} V_{t+1}^{-\alpha} (1+r_{t+1}) C_{t+1}^{-\gamma-1}\, \mathrm{DCDA}_{t+1} \\ & - \alpha \mathrm{VTWIST}_t^{\alpha/(1-\alpha)}\, V_{t+1}^{-\alpha-1} (1+r_{t+1})^2 C_{t+1}^{-2\gamma} \\ & + \alpha \mathrm{VTWIST}_t^{(\alpha/(1-\alpha))-1} \mathrm{V1EXP}_t^2 \,\big] \left[(1+r_t) - (1+w_t\lambda_t)\, \mathrm{DCDA}_t\right], \end{aligned} \tag{A58}$$

$$\mathrm{PDVC}_t \;=\; C_t \;+\; \beta E_t C_{t+1}^{-\gamma} / C_t^{-\gamma} \left(V_{t+1}/\mathrm{VTWIST}_t^{1/(1-\alpha)}\right)^{-\alpha} \mathrm{PDVC}_{t+1}. \tag{A59}$$

$$\mathrm{CRRA}_t \;=\; \mathrm{CARA}_t \, \mathrm{PDVC}_t \,/(1+r_t). \tag{A60}$$

Equation (A55) corresponds to (14), (A56)–(A57) to Proposition 1, and (A59)–(A60) to Definition 2. Equations (11)–(12) are used to rewrite V_1 and V_{11} in terms of derivatives of u. The variable DCDA_t corresponds to $\partial c_t^*/\partial a_t$; equation (A58) is the derivative of (A14) with respect to a_t, which determines how $\partial c_t^*/\partial a_t$ evolves over time. Note that

$$\frac{\partial c_{t+1}^*}{\partial a_t} \;=\; \frac{\partial c_{t+1}^*}{\partial a_{t+1}^*} \left[(1+r_t) - w_t \lambda_t \frac{\partial c_t^*}{\partial a_t} - \frac{\partial c_t^*}{\partial a_t}\right], \tag{A61}$$

which is used in (A58).

We can then solve the system of equations (A46)–(A60) numerically using the Perturbation AIM algorithm of Swanson, Anderson, and Levin (2006) to compute a fifth-order Taylor series approximate solution around the nonstochastic steady state. These nth-order Taylor series approximations are guaranteed to be arbitrarily accurate in a neighborhood of the nonstochastic steady state, but importantly also converge globally within the domain of convergence of the Taylor series as the order of the approximation n becomes large. In practice, the solution seemed to converge globally over the range of values considered for the state variables in Figure 1–5 by about the third or fourth order, so solutions higher than the fifth order are not reported. Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006) solve a standard real business cycle model like (A46)–(A60) using a variety of numerical methods, including second- and fifth-order perturbation methods, and find that the perturbation solutions are among the most accurate methods globally, as well as being the fastest to compute.

[47] These are somewhat more complicated versions of the equations in Swanson (2012), due to the presence of generalized recursive preferences in the present paper.

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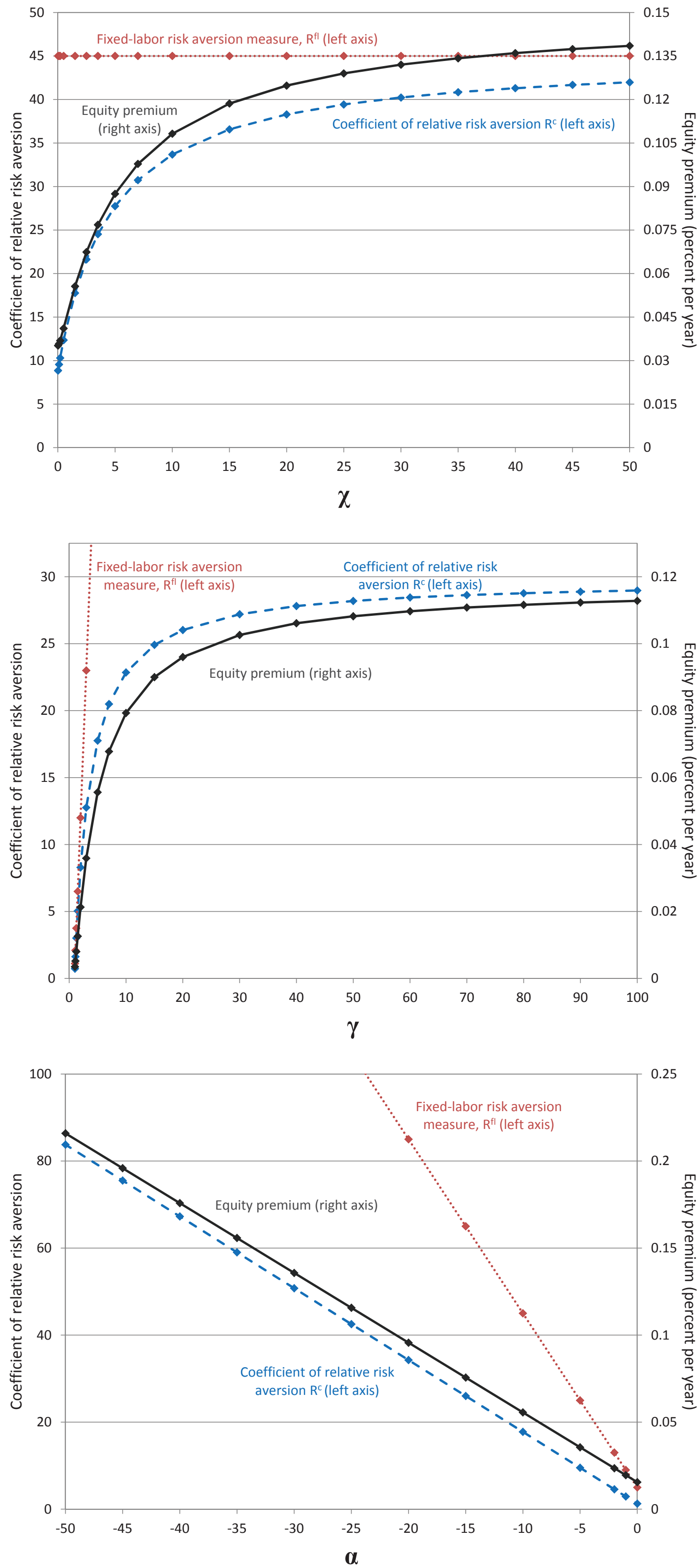

Figure 1. The equity premium and risk aversion in a real business cycle model with generalized recursive preferences and period utility $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l_t^{1+\chi}/(1+\chi)$. Solid black lines depict the equity premium, dashed blue lines the coefficient of relative risk aversion R^c, and dotted red lines the traditional, fixed-labor measure of risk aversion, $R^{fl} = \gamma + \alpha(1-\gamma) = 1 - \widetilde{\alpha}$. In the top panel, χ ranges from .01 to 50 while γ is fixed at 5 and α at -10; in the middle panel, γ ranges from 1.01 to 100 while χ is fixed at 1.5 and α at -10; in the bottom panel, α ranges from -50 to 0 while χ is fixed at 1.5 and γ at 5. In each panel, the equity premium is closely related to R^c and is essentially unrelated to R^{fl}. See text for details.

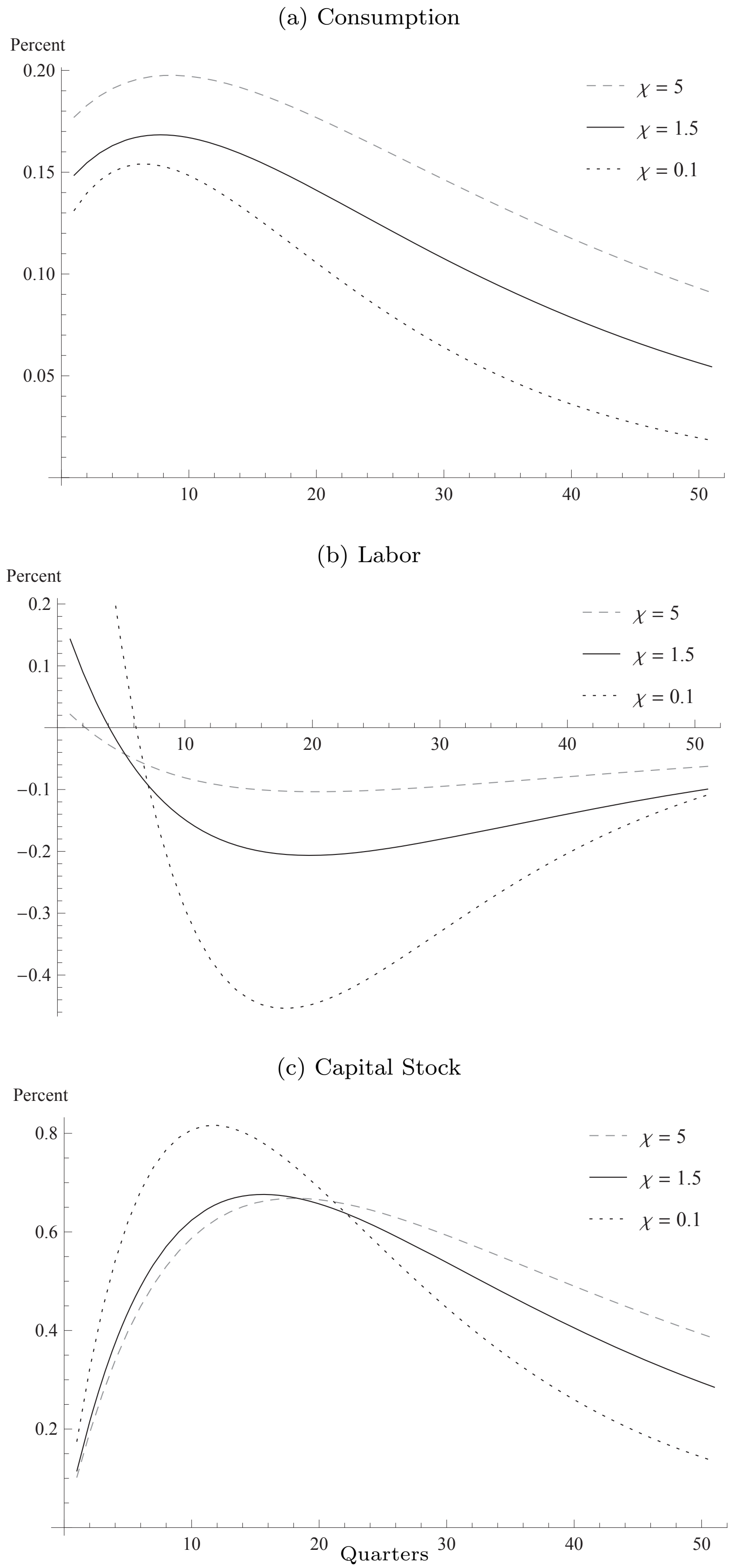

Figure 2. Impulse response functions for (a) consumption, (b) labor, and (c) the capital stock to a 1% technology shock in the real business cycle model from Example 4 and Figure 1, with generalized recursive preferences and period utility $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l_t^{1+\chi}/(1+\chi)$. In each panel, $\gamma = 5$, $\alpha = -10$, and $\chi \in \{0.1, 1.5, 5\}$. When χ is lower, the household varies labor supply by more to smooth consumption, even though labor and consumption comove positively in the short run. See text for details.

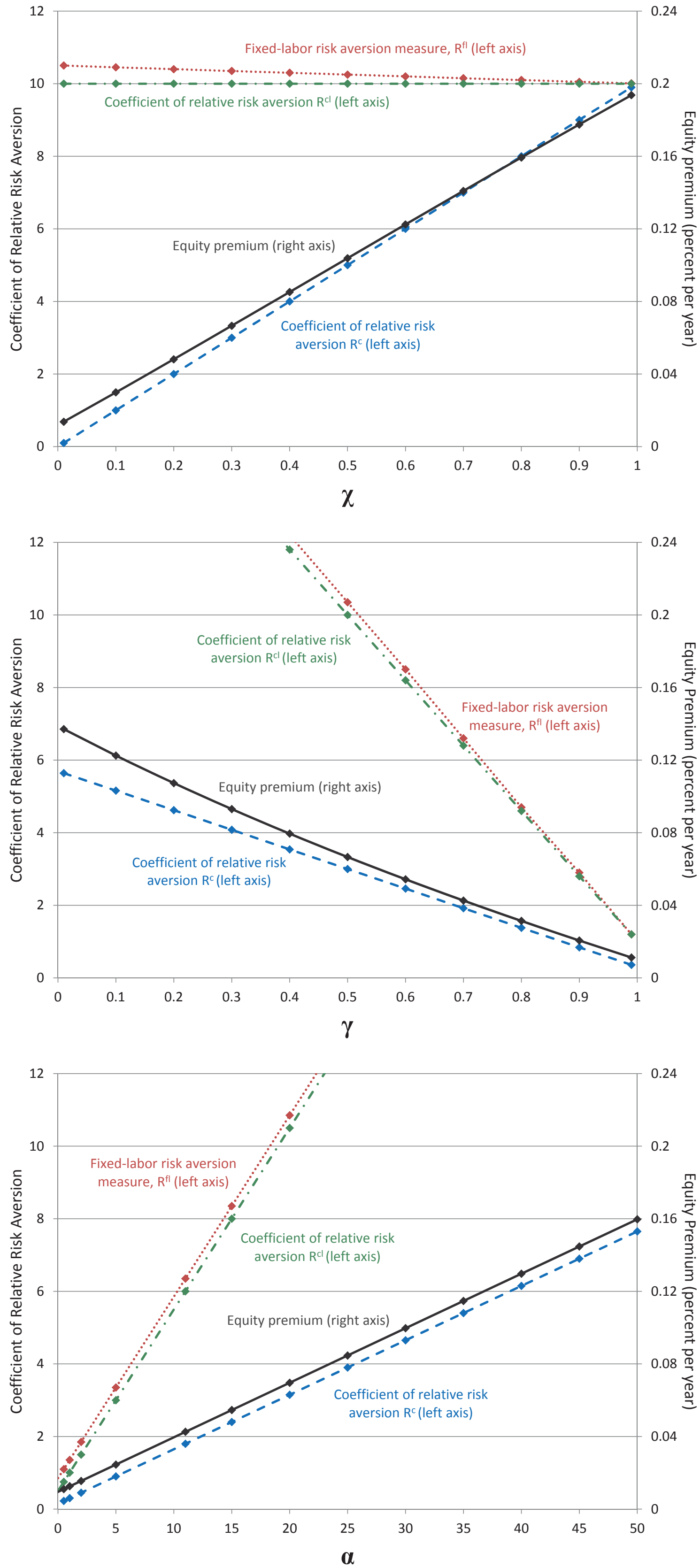

Figure 3. The equity premium and risk aversion in an RBC model with generalized recursive preferences and period utility $u(c_t, l_t) = \big(c_t^{\chi}(1-l_t)^{1-\chi}\big)^{1-\gamma}/(1-\gamma)$. Solid black lines depict the equity premium, dashed blue lines the coefficient of relative risk aversion R^c, dotted red lines the fixed-labor measure of risk aversion R^{fl}, and dash-dot green lines the coefficient of relative risk aversion R^{cl}. In the top panel, χ ranges from .01 to .99 while γ is fixed at 0.5 and α at 19; in the middle panel, γ ranges from .01 to .99 while χ is fixed at 0.3 and α at 19; in the bottom panel, α ranges from 0 to 50 while χ is fixed at 0.3 and γ at 0.5. In each panel, the equity premium is closely related to R^c and is essentially unrelated to R^{fl} and R^{cl}. See text for details.

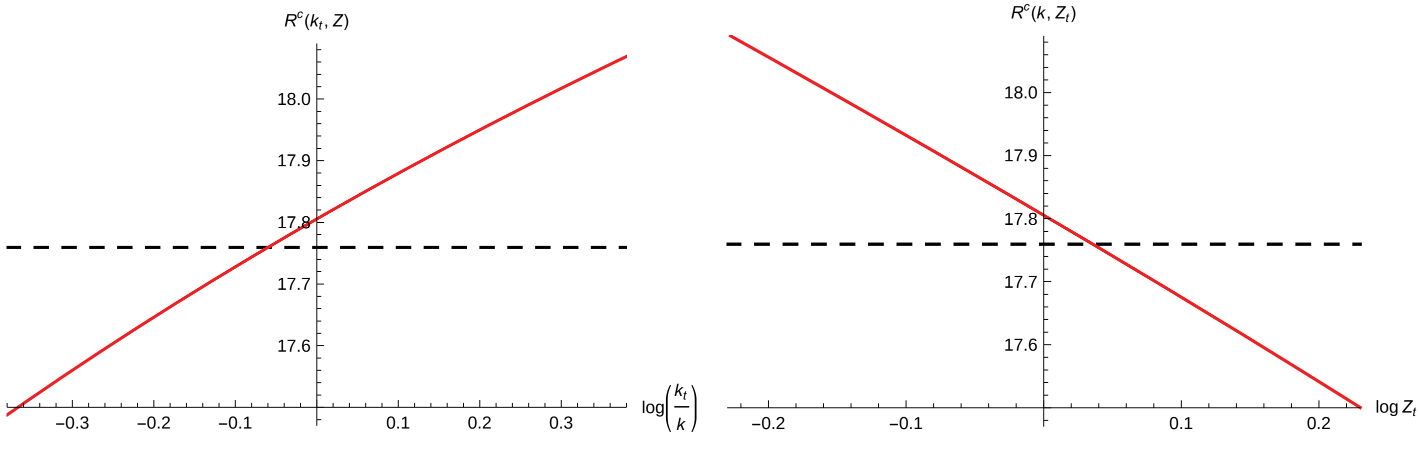

Figure 4. Coefficient of relative risk aversion R^c as a function of the state $(k_t; Z_t)$ in a real business cycle model with generalized recursive preferences and period utility $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l_t^{1+\chi}/(1+\chi)$. Dashed black lines depict the closed-form, steady-state value $R^c(k; Z)$, solid red lines the numerical solution for $R^c(k_t; Z_t)$. In the left panel, $\log(k_t/k)$ ranges from -0.38 to 0.38 while $\log Z_t$ is fixed at 0; in the right panel, Z_t ranges from -0.23 to 0.23 while k_t is fixed at k. In both panels, $R^c(k_t; Z_t)$ is close to $R^c(k; Z)$ and never near the traditional, fixed-labor value of $R^{fl} = 45$. See text for details.

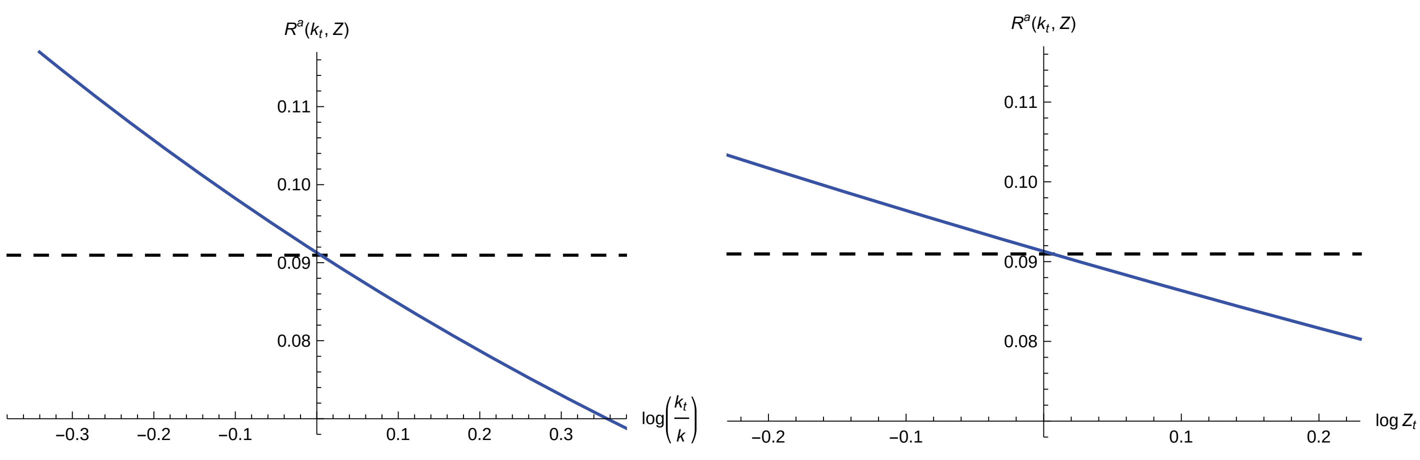

Figure 5. Coefficient of absolute risk aversion R^a as a function of the state $(k_t; Z_t)$ in a real business cycle model with generalized recursive preferences and period utility $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l_t^{1+\chi}/(1+\chi)$. Dashed black lines depict the closed-form, steady-state value $R^a(k; Z)$, solid blue lines the numerical solution for $R^a(k_t; Z_t)$. Absolute risk aversion is decreasing with both k_t and Z_t. See notes to Figure 4 and text for details.