

# Implications of Labor Market Frictions for Risk Aversion and Risk Premia

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## Abstract

A flexible labor margin allows households to absorb shocks to asset values with changes in hours worked as well as changes in consumption. This ability to partially offset asset value shocks by varying hours of work can significantly alter the household's attitudes toward risk, as shown in Swanson (2012). In this paper, I analyze how frictional labor markets affect that analysis. Risk aversion is higher: 1) in countries with more frictional labor markets, 2) in recessions, and 3) for households that have more difficulty finding a job. These predictions are consistent with empirical evidence from a variety of sources. Quantitatively, I show that labor market frictions in Europe are large enough to play a substantial contributing role to risk aversion in those countries. Finally, traditional fixed-labor measures of risk aversion show no stable relationship to the equity premium in a standard real business cycle model with search frictions, while the closed-form expressions I derive here match the equity premium closely.

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## 1. Introduction

A growing macro-finance literature focuses on bringing standard macroeconomic models into better agreement with basic asset pricing facts, such as the equity premium.<sup>1</sup> In asset pricing models, a crucial parameter is risk aversion, the compensation that households require to hold a risky payoff. At the same time, a key feature of standard macroeconomic models is that households have some ability to vary their labor supply. A fundamental difficulty with this line of research, then, is that much of what is known about risk aversion has been derived under the assumption that household labor is exogenously fixed.<sup>2</sup>

Swanson (2012) addresses this problem when households face a frictionless labor market. In the present paper, I re-examine this question in a frictional labor market, as in Mortensen and Pissarides (1994). When the labor market is frictional, the household's ability to partially insure itself from asset value fluctuations by varying hours of work is reduced.

A simple example helps to illustrate the importance of the labor market environment on the household's attitudes toward risk. Suppose that the household has period utility function  $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l_t$ , where  $c_t$  denotes consumption,  $l_t$  hours of work, and  $\gamma, \eta > 0$ . If labor  $l_t$  is exogenously fixed, then the household's coefficient of relative risk aversion is given by the usual  $-c u_{11}/u_1 = \gamma$ . However, when labor is flexible and labor markets are frictionless, the household optimally chooses to insure itself from asset value fluctuations by varying hours of work rather than consumption, resulting in a coefficient of relative risk aversion that is zero (Swanson, 2012)—in other words, the household is essentially *risk neutral* in a frictionless labor market because it optimally varies its labor supply in response to a shock to asset values and is clearly risk-neutral with respect to variations in hours.<sup>3</sup> When labor markets are frictional rather than frictionless, the household's risk aversion lies somewhere in between these two extremes, because its ability to self-insure lies somewhere in between.

In the present paper, I derive closed-form expressions for risk aversion when labor markets

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<sup>1</sup>For example, Boldrin, Christiano, and Fisher (2001), Tallarini (2000), Rudebusch and Swanson (2008, 2012), Uhlig (2007), Van Binsbergen et al. (2012), Backus, Routledge, and Zin (2008), Gourio (2012, 2013), Palomino (2012), Andreasen (2012a, b), Colacito and Croce (2013), Dew-Becker (2014), Kung (2015), and Swanson (2017b) all consider asset pricing in dynamic macroeconomic models with a variable labor margin.

<sup>2</sup>For example, Arrow (1965) and Pratt (1964) define absolute and relative risk aversion,  $-u''(c)/u'(c)$  and  $-c u''(c)/u'(c)$ , in a static model with a single consumption good (and no labor).

<sup>3</sup>This example is extreme, due to the linearity of hours in the household's preferences. More generally, when  $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l_t^{1+\chi}/(1+\chi)$ , Swanson (2012) shows that risk aversion is given by  $(\gamma^{-1} + \chi^{-1})^{-1}$ , a combination of the parameters on the consumption and labor margins, reflecting that the household absorbs shocks to asset values along both margins.

are frictional rather than fixed (the traditional case) or frictionless (as in Swanson, 2012). I show how these closed-form expressions depend on the ratio of labor market flow rates to the household’s discount rate. Intuitively, labor market frictions only delay, and do not prevent, the household’s labor adjustment; thus, higher flow rates or a lower discount rate imply that frictions are less of a concern to the household because this delay is either shorter or less costly.

The closed-form expressions for risk aversion I derive here have three main implications: First, risk aversion is higher in more frictional labor markets (e.g., Continental Europe). Second, risk aversion is higher in recessions, when unemployment is higher and it is more difficult to adjust employment. Third, risk aversion is higher for households that are less likely to find jobs (e.g., retirees, the less educated, and households suffering from discrimination). In all of these cases, it is more difficult for the household to vary its hours in response to shocks, and so more of the burden of asset fluctuations passes through to consumption.

These model predictions are consistent with empirical evidence from a variety of sources. Guiso, Haliassos, and Jappelli (2002) and Ynesta (2008) document that the portfolio holdings of European households are substantially more conservative than those of U.S. households, consistent with the first prediction. Consistent with the second prediction, Guiso, Sapienza, and Zingales (2017) show that survey estimates of household risk aversion increased substantially during the 2008–09 recession. Moreover, Fama and French (1989), Cooper and Priestley (2009), and others show that risk premia on stocks, bonds, and other financial assets tend to be higher in recessions.<sup>4</sup> Consistent with the third prediction, the portfolios of households near retirement are more conservative than those of younger households (Guiso et al., 2002). Although some of these empirical observations may be driven by other factors as well, differences in labor market frictions may play an important contributing role.

I investigate the quantitative importance of these predictions by calibrating the model’s parameters to the U.S. and several European economies. For the U.S., labor market flow rates are sufficiently high that risk aversion is virtually the same as in the frictionless benchmark (Swanson, 2012), both in expansions and recessions. For European countries, differences in labor market frictions can play a significant role in cross-country differences in relative risk aversion, accounting for up to a 35% increase. I show that labor market frictions could play an important role in cyclical variability in risk aversion in these countries as well.

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<sup>4</sup>See also Campbell and Cochrane (1999), Campbell (1999), Lettau and Ludvigson (2010), Piazzesi and Swanson (2008), and Cochrane and Piazzesi (2005).

Finally, these quantitative estimates are conservative, because the only cost of labor market frictions in the model is the cost of delayed labor adjustment, which is completely characterized by the household’s discount rate  $r$ . To the extent that labor market frictions impose additional costs on households through borrowing constraints, depreciation of human capital, or stigma associated with longer spells of unemployment, the costs of labor market frictions will effectively increase  $r$  in the model and lead to larger effects.

## 1.1 Related Literature

Arrow (1965) and Pratt (1964) define absolute and relative risk aversion,  $-u''(c)/u'(c)$  and  $-cu''(c)/u'(c)$ , in terms of the household’s aversion to a fair gamble over a single good in a static, one-period model. When there are multiple goods, the literature has taken two different approaches to generalizing their definition. The first approach, which I follow in this paper, is to consider the household’s aversion to a fair gamble over *money* or *wealth*.<sup>5</sup> This approach is taken by Stiglitz (1969) in a static, multiple-good setting, and by Constantinides (1990), Campbell and Cochrane (1999), and others in a dynamic, single-good setting (in which the multiple goods are a single good consumed at different points in time). As shown in Swanson (2012) and in Section 3, below, this approach implies that the household’s coefficient of “wealth-gamble risk aversion” is related to the curvature of the household’s *indirect* utility function (in the static case) or *value* function (in the dynamic case).

The alternative approach, taken by Kihlstrom and Mirman (1974, 1981), defines a household’s attitudes toward risk solely as a function of its preferences over consumption and its total consumption bundle. Thus, other features of the economic environment that affect the household’s value function but *not* its utility function have no effect on the household’s attitudes toward risk in this approach. When there are multiple goods, the household’s Kihlstrom-Mirman attitudes toward risk are complicated and multi-dimensional, and cannot be summarized by a single “coefficient of risk aversion” except in very special cases. For example, in Kihlstrom and Mirman (1974), one agent cannot be said to be more or less risk averse than another unless both agents have identical ordinal preferences over all nonstochastic consumption bundles. Similarly, in Kihlstrom and Mirman (1981), increasing, constant, and decreasing relative risk aversion are only defined if the agent has preferences that are homothetic.

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<sup>5</sup>Both Arrow (1965) and Pratt (1964) explicitly worked with a “utility function for money” (Pratt, 1964, p. 122) or wealth: “Let  $Y = \text{wealth}$ ,  $U(Y) = \text{total utility of wealth } Y$ ” (Arrow, 1965, pp. 91–92), and defined risk aversion in terms of a fair gamble over money or wealth.

Throughout this paper, I use the first definition of “risk aversion” above, and not the second. The reason for focusing on wealth gambles is asset pricing: a household’s aversion to a wealth gamble is always well-defined under the technical assumptions I make below (e.g., twice-differentiability) and is highly relevant for understanding the household’s aversion to holding a risky asset with monetary payoffs, such as a stock or bond.<sup>6</sup> The Kihlstrom-Mirman (1974, 1981) approach is multi-dimensional in general and provides little insight into how much compensation a household would require to hold a risky asset with monetary payoffs, except in very special cases such as homothetic preferences.<sup>7</sup>

In addition to Swanson (2012), previous studies that consider how the household’s labor margin affects risk aversion include Boldrin, Christiano, and Fisher (1997) and Uhlig (2007). Boldrin et al. consider some very simple endowment economies for which they can compute closed-form expressions for the value function, and hence risk aversion. Uhlig (2007) points out that when households have generalized recursive preferences, leisure affects asset prices because the value function  $V$  appears in the household’s stochastic discount factor, and  $V$  depends on leisure. My analysis here differs from theirs by deriving closed-form solutions for risk aversion in dynamic equilibrium models in general, demonstrating the importance of the labor margin, showing the tight link between risk aversion and asset prices in these models, and showing how labor market frictions affect these results.

There is also a large and growing body of empirical evidence that households vary their labor supply in response to financial shocks—i.e., that the wealth effect on labor supply is negative. Imbens, Rubin, and Sacerdote (2001) and Cesarini, Lundqvist, Notowidigdo, and Östling (2017) find that households who win a prize in the lottery reduce their labor supply significantly, even for moderately-sized prizes. Cesarini et al. (2017) find that this adjustment takes place on both the intensive and extensive margins, and for spouses as well as lottery winners themselves. Coile and Levine (2009) document that older workers are less likely to retire after the stock market performs poorly, and Coronado and Perozek (2003) find that households retire earlier when the stock market performs well. Pencavel (1986) and Killingsworth and Heckman (1986) survey

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<sup>6</sup>The set of assets with monetary payoffs includes essentially all traded assets in the data, even “real” or “indexed” assets, since those assets’ payoffs are transacted in money. Even in the case of a real asset such as an oil well, whose payoffs are in barrels of crude, those payoffs are typically converted into money at a spot rate rather than consumed directly.

<sup>7</sup>Note, however, that when the household has homothetic preferences and faces no constraints other than its budget constraint, the Kihlstrom-Mirman coefficient of risk aversion agrees with the coefficient of wealth-gamble risk aversion that I use in this paper—see Kihlstrom and Mirman (1974, Section 3.1).

estimates of the wealth effect on labor supply and find it to be significantly negative.

The remainder of the paper proceeds as follows. In Section 2, I define a general dynamic equilibrium framework with labor market frictions. In Section 3, I derive closed-form expressions for risk aversion in that framework and presents a numerical example showing the importance of taking the labor margin into account. In Section 4, I derive the implications of labor market frictions for risk aversion described above. I explore the quantitative importance of these results in Section 5. Section 6 concludes. An Appendix provides details of the model, proofs, and numerical solution methods that are outlined in the main text.

## 2. Dynamic Equilibrium Framework with Labor Market Frictions

### 2.1 The Household’s Optimization Problem and Value Function

Time is discrete and continues forever. At each time  $t$ , the household seeks to maximize the expected present discounted value of utility flows,

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} [U(c_\tau) - V(l_\tau + u_\tau)], \quad (1)$$

where  $E_t$  denotes the mathematical expectation conditional on the household’s information set at time  $t$ ,  $\beta \in (0, 1)$  is the discount factor, and  $c_\tau$ ,  $l_\tau$ , and  $u_\tau$  denote the household’s state-contingent plans for future consumption, labor, and unemployment at time  $\tau$ . The explicit state-dependence of these plans is suppressed to reduce notation. Let  $\Omega_c$  denote the domain of  $c_\tau$  and  $\Omega_{lu}$  the set of possible values for  $l_\tau + u_\tau$ .

**Assumption 1.** *The function  $U : \Omega_c \rightarrow \mathbb{R}$  is increasing, twice-differentiable, and strictly concave, and  $V : \Omega_{lu} \rightarrow \mathbb{R}$  is increasing, twice-differentiable, and strictly convex.*

A detailed microfoundation for the household’s preferences in (1) is tangential to the present discussion, but is provided in the Appendix. Briefly, the household consists of a unit continuum of individuals who pool their income. At each time  $t$ , an individual who is not employed can either search for a job or stay home and produce nonmarket goods and services (including “leisure”). The household’s home production function is increasing and concave in the number of individuals staying at home; as a result,  $V$  in (1) is increasing and convex in the number of workers not at home,  $l_t + u_t$ .<sup>8</sup>

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<sup>8</sup>The labor market search literature often assumes that household leisure or home production is linear in the number of workers staying at home (e.g., Shimer, 2010). Assumption 1 requires strict convexity of  $V$  in order to guarantee the uniqueness of the household’s optimal choice of  $(c_t, u_t)$  at each time  $t$ , discussed below. Intuitively, the case of a linear  $V$  can be approximated with a  $V$  having infinitesimal convexity.

The labor market is characterized by search and matching. Household labor  $l_t$  is a state variable rather than a choice variable, evolving according to

$$l_{t+1} = (1 - s)l_t + f(\Theta_t)u_t, \quad (2)$$

where  $s \in [0, 1]$  denotes a constant exogenous rate of job destruction,  $\Theta_t \in \Omega_\Theta$  is a Markovian state vector that is exogenous to the household and characterizes the state of the aggregate economy at time  $t$ , and  $f : \Omega_\Theta \rightarrow [0, 1]$  is a function of the aggregate state that gives the measure of jobs found per unit of unemployed workers searching for a job.

In each period  $t$ , the household chooses  $c_t$  and  $u_t$  (and a state-contingent plan for future  $c_\tau$  and  $u_\tau$ ) to maximize (1), subject to the labor market friction (2), the flow budget constraint

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t, \quad (3)$$

and the no-Ponzi condition

$$\lim_{T \rightarrow \infty} E_t \prod_{\tau=t}^T (1 + r_{\tau+1})^{-1} a_{T+1} \geq 0, \quad (4)$$

where  $a_t$  denotes the household's beginning-of-period assets and  $w_t$ ,  $r_t$ , and  $d_t$  denote the real wage, interest rate, and net transfer payments to the household in each period  $t$ , respectively.

The exogenous Markov state vector  $\Theta_t$  governs the processes for  $w_t$ ,  $r_t$ , and  $d_t$ . Before choosing  $c_t$  and  $u_t$  in each period, the household observes  $\Theta_t$  and hence  $w_t$ ,  $r_t$ , and  $d_t$ . The household's information set and state vector at each date  $t$  is thus  $(a_t, l_t; \Theta_t)$ , where  $a_t$  and  $l_t$  are endogenous and  $\Theta_t$  is exogenous to the household. Let  $X$  denote the domain of  $(a_t, l_t; \Theta_t)$ ,  $\Omega$  the domain of  $(c_t, u_t)$ , and  $\Gamma : X \rightarrow \Omega$  the set-valued correspondence of feasible choices for  $(c_t, u_t)$  for each given  $(a_t, l_t; \Theta_t)$ .

In addition to Assumption 1, a few more technical conditions are required to ensure the value function for the household's optimization problem exists, satisfies the Bellman equation, and is unique, continuous, and concave (see Stokey and Lucas (1990), Alvarez and Stokey (1998), and Rincón-Zapatera and Rodríguez-Palmero (2003) for different sets of such sufficient conditions). The details of these conditions are tangential to the present paper, so I simply assume that:

**Assumption 2.** *There exists a solution  $\mathbb{V} : X \rightarrow \mathbb{R}$  to the household's Bellman equation,*

$$\mathbb{V}(a_t, l_t; \Theta_t) = \max_{(c_t, u_t) \in \Gamma(a_t, l_t; \Theta_t)} U(c_t) - V(l_t + u_t) + \beta E_t \mathbb{V}(a_{t+1}, l_{t+1}; \Theta_{t+1}), \quad (5)$$

where  $l_{t+1}$  is given by (2) and  $a_{t+1}$  by (3), that is unique, continuous, and concave.

The same technical conditions, plus Assumption 1, guarantee the existence of a unique optimal choice for  $(c_t, u_t)$  at each point in time, given  $(a_t, l_t; \Theta_t)$ . Let  $c_t^* \equiv c^*(a_t, l_t; \Theta_t)$  and  $u_t^* \equiv u^*(a_t, l_t; \Theta_t)$  denote the household's optimal choices of  $c_t$  and  $u_t$  as functions of the state  $(a_t, l_t; \Theta_t)$ . Then  $\mathbb{V}$  can be written as

$$\mathbb{V}(a_t, l_t; \Theta_t) = U(c_t^*) - V(l_t^* + u_t^*) + \beta E_t \mathbb{V}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}), \quad (6)$$

where  $a_{t+1}^* \equiv (1+r_t)a_t + w_t l_t^* + d_t - c_t^*$  and  $l_{t+1}^* \equiv (1-s)l_t + f(\Theta_t)u_t^*$ . To avoid boundary solutions for  $c_t^*$  and  $l_t^*$  that would make derivatives and first-order conditions problematic, I require these solutions to be interior:

**Assumption 3.** *For any  $(a_t, l_t; \Theta_t) \in X$ , the household's optimal choice  $(c_t^*, u_t^*)$  exists, is unique, and lies in the interior of  $\Gamma(a_t, l_t; \Theta_t)$ .*

Intuitively, Assumption 3 requires the partial derivatives of  $U$  and  $V$  to grow sufficiently large toward the boundary that only interior solutions for  $c_t^*$  and  $u_t^*$  are optimal for all  $(a_t, l_t; \Theta_t) \in X$ .<sup>9</sup>

Assumptions 1–3 guarantee that  $\mathbb{V}$  is continuously differentiable with respect to  $a$ , but in order to define risk aversion below, I require slightly more than this:

**Assumption 4.** *For any  $(a_t, l_t; \Theta_t)$  in the interior of  $X$ , the second derivative of  $\mathbb{V}$  with respect to its first argument,  $\mathbb{V}_{11}(a_t, l_t; \Theta_t)$ , exists.*

Santos (1991) provides relatively mild sufficient conditions for this assumption to be satisfied; intuitively,  $U$  and  $V$  must be strongly concave. Note that Assumption 4 also implies differentiability of the optimal policy functions,  $c^*$  and  $u^*$ , with respect to  $a_t$ .

## 2.2 Representative Household and Steady State Assumptions

Up to this point, the analysis has focused on a single household in isolation, leaving the other households of the model and the production side of the economy unspecified. Implicitly, the other households and production sector jointly determine the process for  $\Theta_t$  (and hence  $w_t$ ,  $r_t$ , and  $d_t$ ) via general equilibrium, but the assumptions above and much of the analysis below does not depend on those details and can simply treat  $\Theta_t$  as an exogenous Markov process from the point of view of a single household. However, in order to move from more complicated, general

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<sup>9</sup>Interiority is a standard requirement for differentiability of the household's optimal policy functions—see, e.g., Santos (1991).

expressions for risk aversion to more concrete, closed-form expressions, I adopt three standard assumptions from the macroeconomics literature:<sup>10</sup>

**Assumption 5.** *The household is infinitesimal.*

**Assumption 6.** *The household is representative.*

**Assumption 7.** *The model has a nonstochastic steady state, at which  $x_t = x_{t+k}$  for all  $k = 1, 2, \dots$ , and  $x \in \{c, u, l, a, w, r, d, \Theta\}$ .*

Assumption 5 implies that an individual household's choices for  $c_t$  and  $u_t$  have no effect on the aggregate quantities  $w_t$ ,  $r_t$ ,  $d_t$ , and  $\Theta_t$ , determined in general equilibrium. Thus, Assumption 5 explicitly states an assumption that was already implicit in the framework above, in the statement that  $\Theta_t$  (and  $w_t$ ,  $r_t$ , and  $d_t$ ) is exogenous to the household.

Assumption 6 is a restriction on the general equilibrium of the economy: that aggregate consumption demand and labor supply are determined by a household sector that effectively has the same preferences as the given household (either explicitly or through Gorman aggregation). Assumption 6 implies that, when the economy is at the nonstochastic steady state, the given household will find it optimal to choose the steady-state values of  $c$  and  $u$  given  $a$  and  $\Theta$ . (Throughout the text, a variable without a time subscript  $t$  denotes its steady-state value.)<sup>11</sup>

Finally, Assumption 7 is another restriction on the general equilibrium of the economy, in particular on how that equilibrium behaves when there are no aggregate shocks and no aggregate uncertainty.

It's important to note that Assumptions 6–7 do not prohibit offering an individual household a hypothetical gamble of the type described below. The steady state of the model serves only as a reference point around which the *aggregate* variables  $w$ ,  $r$ ,  $d$ , and  $\Theta$  and the *other households'* choices of  $c$ ,  $u$ ,  $a$  and  $l$  can be predicted with certainty. This reference point is important because it is there that closed-form expressions for risk aversion can be computed.

Finally, many dynamic models do not have a steady state *per se*, but rather a balanced growth path, as in King, Plosser, and Rebelo (1988). The results below carry through essentially unchanged to the case of balanced growth. For ease of exposition, Sections 3–5 restrict attention

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<sup>10</sup> Alternative assumptions about the nature of the other households in the model or the production sector may also allow for closed-form expressions for risk aversion. However, the assumptions used here are standard and thus the most natural to pursue.

<sup>11</sup> Let the exogenous state  $\Theta_t$  contain the variances of any shocks to the model, so that  $(a, l; \Theta)$  denotes the nonstochastic steady state, with the variances of any shocks (other than the hypothetical gamble described in the next section) set equal to zero;  $c(a, l; \Theta)$  corresponds to the household's optimal consumption choice at the nonstochastic steady state, etc.

to the case of a steady state, while the Appendix shows the adjustments required under the more general:

**Assumption 7'.** *The model has a balanced growth path that can be renormalized to a non-stochastic steady state after a suitable change of variables.*

### 3. Risk Aversion

#### 3.1 The Coefficient of Absolute Risk Aversion

The household's attitudes toward risk at time  $t$  generally depend on the household's state vector at time  $t$ ,  $(a_t, l_t; \Theta_t)$ . Given this state, I consider the household's aversion to a hypothetical one-shot gamble in period  $t$  of the form

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma \varepsilon_{t+1}, \quad (7)$$

where  $\varepsilon_{t+1}$  is a random variable representing the gamble, with bounded support  $[\underline{\varepsilon}, \bar{\varepsilon}]$ , mean zero, unit variance, independent of  $\Theta_\tau$  for all times  $\tau$ , and independent of  $a_\tau, l_\tau, c_\tau$ , and  $u_\tau$  for all  $\tau \leq t$ . A few words about (7) are in order: First, the gamble is dated  $t + 1$  to clarify that its outcome is not in the household's information set at time  $t$ . Second,  $c_t$  cannot be made the subject of the gamble without substantial modifications to the household's optimization problem, because  $c_t$  is a choice variable under control of the household at time  $t$ . However, (7) is clearly equivalent to a one-shot gamble over net transfers  $d_t$  or asset returns  $r_t$ , both of which are exogenous to the household.<sup>12</sup> Indeed, thinking of the gamble as being over  $r_t$  helps to illuminate the connection between (7) and the price of risky assets, which I will discuss further in Section 3.3, below. As shown there, the household's aversion to the gamble in (7) is directly linked to the premium households require to hold risky assets.

Following Arrow (1964) and Pratt (1965), I ask what one-time fee  $\mu$  the household would be willing to pay in period  $t$  to avoid the gamble in (7):

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu. \quad (8)$$

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<sup>12</sup>In this case, the realized transfer  $d_t + \sigma \varepsilon_{t+1}$ , or asset return  $r_t + \sigma \varepsilon_{t+1}$ , would not be in the household's time- $t$  information set,  $(a_t; \theta_t)$ .

The quantity  $\mu$  that makes the household just indifferent between (7) and (8), for infinitesimal  $\sigma$  and  $\mu$ , is the household's coefficient of absolute risk aversion. Formally, this corresponds to the following definition:

**Definition 1.** Let  $(a_t, l_t; \Theta_t)$  be an interior point of  $X$ . Let  $\tilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma)$  denote the value function for the household's optimization problem inclusive of the one-shot gamble (7), and let  $\mu(a_t, l_t; \Theta_t; \sigma)$  denote the value of  $\mu$  that satisfies  $\mathbb{V}(a_t - \frac{\mu}{1+r_t}; \Theta_t) = \tilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma)$ . The household's coefficient of absolute risk aversion at  $(a_t, l_t; \Theta_t)$ , denoted  $R^a(a_t, l_t; \Theta_t)$ , is given by  $R^a(a_t, l_t; \Theta_t) = \lim_{\sigma \rightarrow 0} \mu(a_t, l_t; \Theta_t; \sigma) / (\sigma^2/2)$ .

In Definition 1,  $\mu(a_t, l_t; \Theta_t; \sigma)$  denotes the household's "willingness to pay" to avoid a one-shot gamble of size  $\sigma$  in (7). As in Arrow (1964) and Pratt (1965),  $R^a$  denotes the limit of the household's willingness to pay per unit of variance as this variance becomes small. Note that  $R^a(a_t, l_t; \Theta_t)$  depends on the economic state because  $\mu(a_t, l_t; \Theta_t; \sigma)$  depends on that state. Proposition 1 shows that  $\tilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma)$ ,  $\mu(a_t, l_t; \Theta_t; \sigma)$ , and  $R^a(a_t, l_t; \Theta_t)$  in Definition 1 are well-defined and that  $R^a(a_t, l_t; \Theta_t)$  equals the "folk wisdom" value of  $-\mathbb{V}_{11}/\mathbb{V}_1$ .<sup>13</sup>

**Proposition 1.** Let  $(a_t, l_t; \Theta_t)$  be an interior point of  $X$ . Given Assumptions 1–5,  $\tilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma)$ ,  $\mu(a_t, l_t; \Theta_t; \sigma)$ , and  $R^a(a_t, l_t; \Theta_t)$  exist and

$$R^a(a_t, l_t; \Theta_t) = \frac{-E_t \mathbb{V}_{11}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1})}{E_t \mathbb{V}_1(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1})}, \quad (9)$$

where  $\mathbb{V}_1$  and  $\mathbb{V}_{11}$  denote the first and second partial derivatives of  $\mathbb{V}$  with respect to its first argument. Given Assumptions 6–7, (9) can be evaluated at the steady state to yield

$$R^a(a, l; \Theta) = \frac{-\mathbb{V}_{11}(a, l; \Theta)}{\mathbb{V}_1(a, l; \Theta)}. \quad (10)$$

PROOF: See Appendix.

Equation (9) is valid quite generally and requires only the technical Assumptions 1–5 on preferences and that  $\theta_t$  is exogenous to the household. However, (9) can only be solved numerically in general; equation (10) is much simpler (and can be simplified further, below), which shows the benefits of assuming a representative agent nonstochastic steady state to use as a reference point.

Definition 1 and Proposition 1 imply that risk aversion is related to the curvature of the value function  $\mathbb{V}$  with respect to wealth rather than the curvature of  $U$  with respect to consumption. Because the value function depends on the economic state  $\Theta_t$  (and, implicitly, on the

<sup>13</sup>See, e.g., Constantinides (1990), Farmer (1990), Campbell and Cochrane (1999), and Flavin and Nakagawa (2008). For the more general case of Epstein-Zin (1990) preferences, equation (9) no longer holds and there is no folk wisdom; see Swanson (2017a) for the more general expressions corresponding to that case.

stochastic process governing  $\Theta_t$ ), risk aversion also depends on  $\Theta_t$  (and its stochastic process). This state-dependence is standard in the literature on wealth-gamble risk aversion: for example, Stiglitz (1969) defines risk aversion in terms of a fair gamble over money or wealth, with absolute and relative risk aversion of  $-U_{yy}/U_y$  and  $-yU_{yy}/U_y$ , where the indirect utility function  $U$  “is a function of both [income]  $y$  and [prices]  $p$ :  $U = U(y, p)$ . Arrow and Pratt assumed in effect that  $p$  is constant” (Stiglitz, 1969, footnote 11). In Stiglitz (1969), risk aversion clearly depends on the economic state—the price vector  $p$ —as well as the agent’s preferences and total consumption bundle. Constantinides (1990) and Campbell and Cochrane (1999) work in a dynamic framework, where the household’s aversion to money or wealth gambles implies that it is the *value* function that is relevant, as in the present paper: “I define the RRA coefficient in terms of an atemporal gamble that changes the current level of capital by the outcome of the gamble. . .  $RRA = -WV_{ww}/V_w$  . . . The RRA coefficient is a function of wealth and of the state variables  $x(t)$ ,” (Constantinides, 1990, p. 527); and “Risk aversion measures attitudes toward pure wealth bets and is therefore conventionally captured by the second partial derivative of the value function with respect to individual wealth. . .” (Campbell and Cochrane, 1999, p. 243), and risk aversion “depends on individual wealth  $W$  and on aggregate variables that describe asset prices or investment opportunities. . .” (Campbell and Cochrane, 1999, p. 244).<sup>14</sup> Note that this state-dependence is not a feature of the Kihlstrom-Mirman (1974, 1981) approach to risk aversion mentioned in the Introduction; that approach defines the household’s attitudes toward risk in a multi-dimensional way that is independent of the economic state (and its stochastic process) and depends only on the household’s preferences and total consumption bundle.

A practical difficulty with Proposition 1 is that closed-form expressions for the value function  $\mathbb{V}$  do not exist in general, even for the simplest dynamic models with labor. I solve this problem by observing that  $\mathbb{V}_1$  and  $\mathbb{V}_{11}$  often can be computed even when closed-form solutions for  $\mathbb{V}$  cannot be. For example, the Benveniste-Scheinkman equation,

$$\mathbb{V}_1(a_t, l_t; \Theta_t) = (1 + r_t) U'(c_t^*), \quad (11)$$

states that the marginal value of a dollar of assets equals the marginal utility of consumption times  $1 + r_t$  (the interest rate appears here because beginning-of-period assets in the model generate income in period  $t$ ). In (11),  $U'$  is a known function. Although a closed-form solution for the

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<sup>14</sup>See also Farmer (1990) and Flavin and Nakagawa (2008), who use essentially the same definition and come to the same conclusions in their dynamic frameworks.

function  $c^*$  is not known in general, the point  $c_t^*$  often is known—for example, when it is evaluated at the nonstochastic steady state,  $c$ . Thus, we can compute  $\mathbb{V}_1$  at the nonstochastic steady state by evaluating the right-hand side of (11) at that point.

The second derivative  $\mathbb{V}_{11}$  can be computed by noting that equation (11) holds for general  $a_t$ ; hence we can differentiate it to yield

$$\mathbb{V}_{11}(a_t, l_t; \Theta_t) = (1 + r_t) U''(c_t^*) \frac{\partial c_t^*}{\partial a_t}. \quad (12)$$

All that remains is to find the derivative  $\partial c_t^*/\partial a_t$ .

Intuitively,  $\partial c_t^*/\partial a_t$  should not be too difficult to compute: it is just the household's marginal propensity to consume today out of a change in assets, which can be deduced from the household's Euler equation and budget constraint. Differentiating the Euler equation

$$U'(c_t^*) = \beta E_t(1 + r_{t+1}) U'(c_{t+1}^*) \quad (13)$$

with respect to  $a_t$  yields<sup>15</sup>

$$U''(c_t^*) \frac{\partial c_t^*}{\partial a_t} = \beta E_t(1 + r_{t+1}) U''(c_{t+1}^*) \frac{\partial c_{t+1}^*}{\partial a_t}. \quad (14)$$

Evaluating (14) at steady state,  $\beta = (1 + r)^{-1}$  and the  $U''(c)$  factors cancel, giving

$$\frac{\partial c_t^*}{\partial a_t} = E_t \frac{\partial c_{t+1}^*}{\partial a_t} = E_t \frac{\partial c_{t+k}^*}{\partial a_t}, \quad k = 1, 2, \dots \quad (15)$$

In other words, starting from steady state, whatever the change in the household's optimal consumption today, it must be the same as the change in the household's expected optimal consumption tomorrow, and the change in the household's expected optimal consumption at each future date  $t + k$ .<sup>16</sup>

The household's budget constraint is implied by asset accumulation equation (3) and the no-Ponzi condition (4). Differentiating (3) with respect to  $a_t$ , evaluating at steady state, and applying (4) gives

$$\sum_{k=0}^{\infty} \frac{1}{(1 + r)^k} E_t \left[ \frac{\partial c_{t+k}^*}{\partial a_t} - w \frac{\partial l_{t+k}^*}{\partial a_t} \right] = 1 + r. \quad (16)$$

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<sup>15</sup>The notation  $\frac{\partial c_{t+1}^*}{\partial a_t}$  is taken to mean  $\frac{\partial c_{t+1}^*}{\partial a_{t+1}} \frac{da_{t+1}^*}{da_t} + \frac{\partial c_{t+1}^*}{\partial l_{t+1}} \frac{dl_{t+1}^*}{da_t} = \frac{\partial c_{t+1}^*}{\partial a_{t+1}} \left[ (1 + r_t) - \frac{\partial c_t^*}{\partial a_t} \right] + \frac{\partial c_{t+1}^*}{\partial l_{t+1}} f(\Theta_t) \frac{\partial u_t}{\partial a_t}$ , and analogously for  $\frac{\partial c_{t+2}^*}{\partial a_t}$ ,  $\frac{\partial c_{t+3}^*}{\partial a_t}$ , etc.

<sup>16</sup>Note that this equality does not follow from the steady state assumption. For example, in a model with internal habits, considered in Swanson (2009), the individual household's optimal consumption response to a change in assets increases with time, even starting from steady state.

In other words, the present discounted value of the change in consumption equals the change in assets plus the present discounted value of the change in labor income.

To solve for  $\partial c_t/\partial a_t$  using equations (15)–(16), it only remains to solve for  $\partial l_{t+k}^*/\partial a_t$ . I do this in two steps, using the household's Euler equation for unemployment and the transition equation (2) for labor. The details of this computation are tangential to the main points of this section, so I summarize the result in the following lemma:

**Lemma 1.** *Given Assumptions 1–7 and either  $s < 1$  or  $f(\Theta) < 1$ , the household's expected marginal propensity to work at each future date  $t + k$ ,  $k = 1, 2, \dots$ , with respect to changes in assets at time  $t$ , evaluated at steady state, satisfies*

$$E_t \frac{\partial l_{t+k}^*}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l+u}{c} \frac{f(\Theta)}{s+f(\Theta)} [1 - (1-s-f(\Theta))^k] \frac{\partial c_t^*}{\partial a_t}, \quad (17)$$

where  $\gamma \equiv -cU''(c)/U'(c)$  is the elasticity of  $U'$  with respect to  $c$ , evaluated at steady state, and  $\chi \equiv (l+u)V''(l+u)/V'(l+u)$  the elasticity of  $V'$  with respect to  $l+u$ , evaluated at steady state.

PROOF: See Appendix.

Note that, in response to a change in assets, household consumption jumps instantly to a new steady-state level, but  $l$  responds only gradually, approaching a new steady-state level asymptotically as  $k \rightarrow \infty$ . The household adjusts along the labor margin by relatively more when  $\chi$  is low (i.e., the marginal disutility of working is flat),  $\gamma$  is high (the marginal utility of consumption is curved), or the probability of finding a job  $f(\Theta)$  is high.

Substituting (17) into the budget constraint (16) and solving for  $\partial c_t^*/\partial a_t$  yields

$$\frac{\partial c_t^*}{\partial a_t} = \frac{r}{1 + w \frac{\gamma}{\chi} \frac{l+u}{c} \frac{f(\Theta)}{r+s+f(\Theta)}}. \quad (18)$$

In response to a unit increase in assets, the household raises consumption in every period by the extra asset income,  $r$  (essentially the permanent income hypothesis), adjusted downward by  $1 + w \frac{\gamma}{\chi} \frac{l+u}{c} \frac{f(\Theta)}{r+s+f(\Theta)}$ , which takes into account the household's decrease in hours worked and labor income. Thus, equation (18) represents a modified permanent income hypothesis that takes into account the household's labor margin. When  $f(\Theta)$  is large relative to  $r+s$ , (18) converges to the expression derived in Swanson (2012) for a frictionless labor market. Alternatively, when  $f(\Theta) = 0$ , labor is exogenously fixed and (18) equals  $r$ , the traditional permanent income hypothesis value.

We can now write the household's coefficient of absolute risk aversion in terms of known quantities. Substituting (11), (12), and (18) into (10) proves the following:

**Proposition 2.** *Given Assumptions 1–7, the household's coefficient of absolute risk aversion,  $R^a(a_t, l_t; \Theta_t)$ , evaluated at steady state, satisfies*

$$R^a(a, l; \Theta) = \frac{-U''(c)}{U'(c)} \frac{r}{1 + w \frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\Theta)}{r + s + f(\Theta)}}. \quad (19)$$

There are several features of Proposition 2 worth noting. If labor supply is exogenously fixed, corresponding to  $s = f(\Theta) = 0$ , then risk aversion in (19) reduces to  $-rU''/U'$ , the usual Arrow-Pratt definition multiplied by a scale factor  $r$ , which translates assets into units of current-period consumption.<sup>17</sup> More generally, when  $f(\Theta) > 0$ , households can partially offset shocks to asset values through changes in hours worked. Even though consumption and labor are additively separable in (1), the household's consumption process is still connected to the labor market through the household's budget constraint. As a result, the household's aversion to a gamble over assets is related to its ability to offset asset fluctuations by varying hours of work.

A flexible labor margin implies that risk aversion is less than in the fixed-labor case:

**Corollary 1.** *The coefficient of absolute risk aversion,  $R^a(a_t, l_t; \Theta_t)$ , satisfies*

$$R^a(a, l; \Theta) \leq \frac{-rU''(c)}{U'(c)}. \quad (20)$$

If  $r < 1$ , then (18) is also less than  $-U''/U'$ .

Note that, since  $r$  is the net interest rate,  $r \ll 1$  in typical calibrations.

I discuss the relationship between labor market flexibility, risk aversion, and risk premia below, after first defining relative risk aversion.

### 3.2 The Coefficient of Relative Risk Aversion

The distinction between absolute and relative risk aversion lies in the size of the hypothetical gamble faced by the household. If the household faces a one-shot gamble of size  $A_t$  in period  $t$ ,

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + A_t \sigma \varepsilon_{t+1}, \quad (21)$$

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<sup>17</sup>A gamble over a lump sum of  $\$x$  is equivalent here to a gamble over an annuity of  $\$rx$ . Thus, even though  $\mathbb{V}_{11}/\mathbb{V}_1$  is different from  $U''/U'$  by a factor of  $r$ , this difference is exactly the same as a change from lump-sum to annuity units. Thus, the difference in scale is just one of units.

or the household can pay a one-time fee  $A_t\mu$  in period  $t$  to avoid this gamble, then it follows from Proposition 1 that  $\lim_{\sigma \rightarrow 0} 2\mu(\sigma)/\sigma^2$  for this gamble is given by

$$\frac{-A_t E_t \mathbb{V}_{11}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1})}{E_t \mathbb{V}_1(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1})}. \quad (22)$$

The natural definition of  $A_t$ , considered by Arrow (1964) and Pratt (1965), is the household's wealth at time  $t$ . The gamble in (21) is then over a fraction of the household's wealth and (22) is referred to as the household's coefficient of relative risk aversion.

In models with labor, however, household wealth can be more difficult to define because of the presence of human capital (see Swanson, 2012, 2017a).<sup>18</sup> These issues are tangential to the present paper, so for simplicity I define human capital here to be the present discounted value of labor earnings, as suggested by the results in Swanson (2017a). Equivalently, from the budget constraint (3)–(4), household wealth equals the present discounted value of consumption.

**Definition 2.** *Let  $(a_t, l_t; \Theta_t)$  be an interior point of  $X$ . The household's coefficient of relative risk aversion, denoted  $R^c(a_t, l_t; \Theta_t)$ , is given by (22) with wealth  $A_t \equiv (1+r_t)^{-1} E_t \sum_{\tau=t}^{\infty} m_{t,\tau} c_{\tau}^*$ , the present discounted value of household consumption, where  $m_{t,\tau} = \beta U'(c_{\tau})/U'(c_t)$  denotes the household's stochastic discount factor.*

The factor  $(1+r_t)^{-1}$  in the definition expresses wealth  $A_t$  in beginning- rather than end-of-period- $t$  units, so that in steady state  $A = c/r$  and relative risk aversion is given by

$$R^c(a, l; \Theta) = \frac{-A \mathbb{V}_{11}(a, l; \Theta)}{\mathbb{V}_1(a, l; \Theta)} = \frac{\gamma}{1 + w \frac{\gamma}{\chi} \frac{l+u}{c} \frac{f(\Theta)}{r+s+f(\Theta)}}, \quad (23)$$

where (23) makes use of the definition  $\gamma = -cU''(c)/U'(c)$ . Note that if labor is exogenously fixed, so that  $s = f(\Theta) = 0$ , equation (23) reduces to the usual Arrow-Pratt definition. But as long as the household has some ability to vary its hours of work, risk aversion is reduced by the factor in the denominator of (23).

### 3.3 Numerical Example

The relationship between the labor margin, risk aversion, and risk premia can be seen in a simple real business cycle model with labor market frictions.<sup>19</sup> Let the economy consist of a

<sup>18</sup>Note that the household's financial assets  $a_t$  are *not* a good measure of wealth  $A_t$ , since  $a_t$  for an individual household may be zero or negative at some points in time.

<sup>19</sup>Swanson (2012, 2017a) compares risk aversion to the equity premium in several examples where households can vary their labor supply in a frictionless labor market.

unit continuum of representative households, each with optimization problem (1)–(4) and period utility function

$$U(c_t) - V(l_t + u_t) \equiv \frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{(l_t + u_t)^{1+\chi}}{1+\chi}. \quad (24)$$

There is a unit continuum of perfectly competitive firms, each with production function

$$y_t = Z_t k_t^{1-\alpha} l_t^\alpha, \quad (25)$$

where  $y_t$ ,  $l_t$ , and  $k_t$  denote firm output, labor, and beginning-of-period capital, respectively, and  $Z_t$  denotes an exogenous aggregate productivity process that follows

$$\log Z_t = \rho \log Z_{t-1} + \varepsilon_t. \quad (26)$$

The innovations  $\varepsilon_t$  in (26) are i.i.d. with mean zero and variance  $\sigma_\varepsilon^2$ . Firms rent capital from households in a frictionless competitive market at rental rate  $r_t^k$ . Households accumulate capital according to

$$k_{t+1} = (1 + r_t)k_t + w_t l_t - c_t, \quad (27)$$

where  $r_t = r_t^k - \delta$  and  $\delta$  denotes the capital depreciation rate.

Firms hire labor by posting vacancies  $v_t$  at a cost of  $\kappa$  per vacancy per period. The number of workers employed by each firm evolves according to

$$l_{t+1} = (1-s)l_t + h_t, \quad (28)$$

where  $l_t$  is the number of workers employed by the firm and  $h_t$  the number of new hires.<sup>20</sup> New hires are determined by the Cobb-Douglas matching function,<sup>21</sup>

$$h_t = \mu u_t^{1-\eta} v_t^\eta. \quad (29)$$

This implies the job-finding rate for households is

$$f(\Theta_t) = \frac{h_t}{u_t} = \mu \left( \frac{v_t}{u_t} \right)^\eta. \quad (30)$$

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<sup>20</sup>Note that both firms and households are representative and have unit measure, so the number of workers employed by each firm and the number of household members who work is given by  $l_t$ .

<sup>21</sup>Some authors interpret the Cobb-Douglas matching function in (29) to be  $h_t = \max\{\mu u_t^{1-\eta} v_t^\eta, u_t\}$  so that  $h_t \leq u_t$ . However,  $h_t \leq u_t$  holds around the steady state in this example, so including this max operator does not affect numerical solutions local to the model's steady state.

As is typical in these models, the job-finding rate depends only on the vacancy-unemployment ratio,  $v_t/u_t$ , which is often denoted by  $\theta_t$ . In the present paper, the aggregate state vector  $\Theta_t$  is more general than this, but  $f(\Theta_t)$  nevertheless depends only on  $v_t/u_t$  in this example.

At the beginning of each period  $t$ , workers and firms who were matched in the previous period bargain over the wage  $w_t$ . If negotiations break down, the worker and firm each can search for a new match in period  $t$ . Let  $J_t$  denote the representative firm's surplus from hiring an additional worker in period  $t$ :

$$J_t = \alpha \frac{y_t}{l_t} - w_t + (1-s)E_t m_{t+1} J_{t+1}. \quad (31)$$

The firm's surplus is the difference between the marginal product of labor and the wage this period, plus the expected discounted value of the firm surplus next period, if the match persists.

Let  $S_t$  denote the representative household's marginal surplus from employment,

$$S_t = w_t + (1-s-f(\Theta_t))E_t m_{t+1} S_{t+1}. \quad (32)$$

The household's surplus, relative to being unemployed, is the wage plus the expected discounted value of the surplus next period, if the match persists. (For simplicity, I assume that there is no compensation for being unemployed; also note that the household incurs marginal disutility  $V'(l_t + u_t)$  whether the individual works or is unemployed.)<sup>22</sup>

The wage  $w_t$  in each period is set by Nash bargaining, so that

$$(1-\nu)S_t = \nu J_t, \quad (33)$$

where  $\nu \in [0, 1]$  denotes the household's Nash bargaining weight.

In equilibrium, the marginal cost and marginal benefit to the firm of hiring a worker are equal, so

$$J_t = \kappa \frac{v_t}{h_t} = \frac{\kappa}{\mu} \left( \frac{v_t}{u_t} \right)^{1-\eta}. \quad (34)$$

Similarly, the marginal cost and benefit to the household of searching for a job are equal, giving the household's unemployment Euler equation (equation (A14) in the Appendix).

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<sup>22</sup>Let  $\mathbb{V}_t^E$ ,  $\mathbb{V}_t^U$ , and  $\mathbb{V}_t^H$  denote the value to the household of an individual being employed, unemployed, and at home, respectively. Then  $\mathbb{V}_t^E = w_t - V'(l_t + u_t)/U'(c_t) + (1-s)E_t m_{t+1} \mathbb{V}_{t+1}^E + sE_t m_{t+1} \mathbb{V}_{t+1}^U$ . Because there is no compensation for unemployment,  $\mathbb{V}_t^U = -V'(l_t + u_t)/U'(c_t) + f(\Theta_t)E_t m_{t+1} \mathbb{V}_{t+1}^E + (1-f(\Theta_t))E_t m_{t+1} \mathbb{V}_{t+1}^U$ . Thus  $S_t = \mathbb{V}_t^E - \mathbb{V}_t^U$ . Note that individuals can move freely between unemployment and home production, so  $\mathbb{V}_t^U = \mathbb{V}_t^H$ . Thus,  $\mathbb{V}_t^H$  does not need to be computed to derive  $S_t$ , although  $\mathbb{V}_t^H = H'(h_t) + E_t m_{t+1} \mathbb{V}_t^H$ , using the notation in the Appendix. It follows that  $\mathbb{V}_t^H > 0$ ,  $\mathbb{V}_t^U > 0$ , and  $\mathbb{V}_t^E > \mathbb{V}_t^U$ .

TABLE 1: BENCHMARK PARAMETER VALUES

$\beta$	.996	$\alpha$	.7	$s$	.02
$\gamma$	200	$\rho$	.98	$\eta$	.5
$\chi$	200	$\sigma_\varepsilon$	.005	$\nu$	.5
		$\delta$	.0028		

The numerical example is calibrated to monthly data;  $\chi_0$ ,  $\kappa$ , and  $\mu$  are set to achieve steady-state values of  $l + u = 0.3$ ,  $v/u = 0.6$ , and  $f(\Theta) = 0.28$ , respectively. Benchmark values for  $\gamma$  and  $\chi$  are high in order to achieve a nontrivial equity premium, but a wide range for these parameter values is considered in Figure 1, below. See text for details.

As discussed in Shimer (2010), models with frictional labor markets are more naturally calibrated to monthly than to quarterly data, because unemployed workers in the U.S. typically find jobs in much less than one quarter. Benchmark values for the model's parameters are reported in Table 1. I set the household's discount factor  $\beta$  to .996, as in Shimer (2010), implying an annual real interest rate of about 5 percent. The utility curvature parameters with respect to consumption and labor,  $\gamma$  and  $\chi$ , are each set to 200 in order to generate a nontrivial equity premium in the model, but I also consider much lower values for these parameters in Figure 1, below. I set the utility parameter  $\chi_0$  governing the disutility of work to achieve a target value of  $l + u = 0.3$  in steady state; that is, the household is assumed to devote about 30 percent of its time endowment to market work and labor market search.

I calibrate labor's share of output,  $\alpha$ , to 0.7. The exogenous productivity process is assumed to have a monthly persistence  $\rho = 0.98$  and a shock standard deviation of  $\sigma_\varepsilon = .005$ , as in Shimer (2010). I set the capital depreciation rate  $\delta$  to .0028, also following Shimer (2010), and implying a steady-state capital/annual output ratio of 3.2.

Following Shimer (2010), I set the exogenous job separation rate  $s$  to .02, the wage bargaining parameter  $\nu$  to 0.5, and the matching function elasticity  $\eta$  to 0.5. Firms' cost of posting a vacancy  $\kappa$  is set to achieve a target ratio  $v/u = 0.6$  in steady state, consistent with the estimates in den Haan, Ramey, and Watson (2000) and Hall (2005). I set the matching function productivity parameter  $\mu$  to achieve a target of  $f(\Theta) = 0.28$  in steady state, consistent with the estimate in Shimer (2012).

An equity security in the model is defined to be a claim on the aggregate consumption stream, where aggregate consumption  $C_t = c_t$  in equilibrium. The ex-dividend price of the equity claim,  $p_t$ , satisfies

$$p_t = E_t m_{t+1} (C_{t+1} + p_{t+1}) \quad (35)$$

in equilibrium, where  $m_{t+1} = \beta c_t^\gamma / c_{t+1}^\gamma$  denotes the household's stochastic discount factor. The equity premium,  $\psi_t$ , is defined to be the expected excess return,

$$\psi_t \equiv \frac{E_t(C_{t+1} + p_{t+1})}{p_t} - (1 + r_t^f), \quad (36)$$

where  $(1 + r_t^f) \equiv 1/(E_t m_{t+1})$  denotes the one-period gross risk-free interest rate.

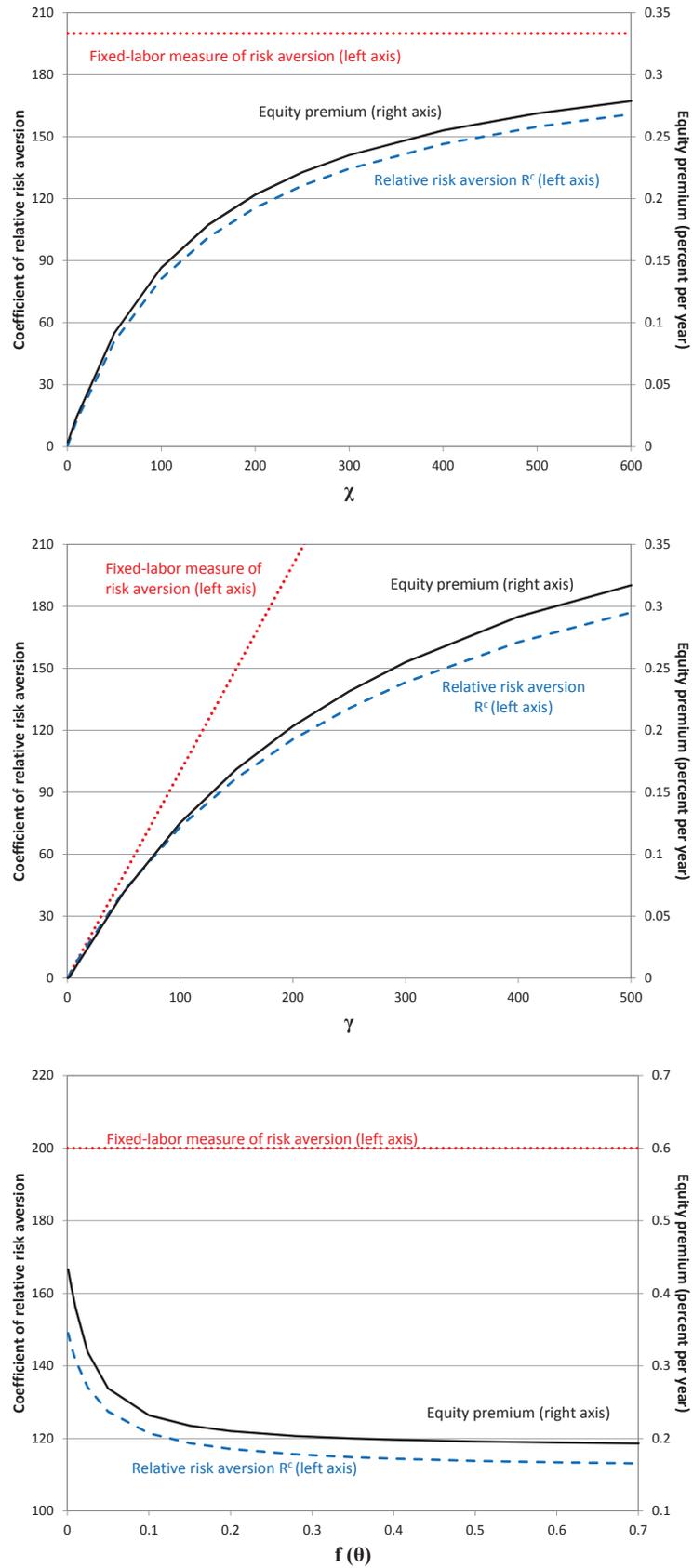
For any given set of parameter values, the model is solved numerically using perturbation methods, as in Swanson (2012). This involves computing a nonstochastic steady state for the model and an  $n$ th-order Taylor series approximation to the true nonlinear solution for the model's endogenous variables around the steady state. (Results in the figures below are for a fifth-order approximation,  $n = 5$ .) The numerical algorithm is described in more detail in Swanson (2012) and Swanson, Anderson, and Levin (2006). Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006) solve a standard RBC model using a variety of numerical methods and find that the fifth-order perturbation solution is among the most accurate globally as well as being the fastest to compute.

Figure 1 graphs the equity premium and risk aversion as functions of  $\chi$ ,  $\gamma$ , and  $f(\Theta)$ , holding the values of the other model parameters fixed at their benchmark values in Table 1. In the top panel,  $\chi$  ranges from 0.5 to 600; in the middle panel,  $\gamma$  varies from 0.5 to 500; and in the bottom panel,  $f(\Theta)$  varies from 0.001 to 0.7 (which is achieved by varying the matching function productivity parameter  $\mu$ ). In each panel, the dotted red line graphs the traditional, fixed-labor measure of risk aversion,  $\gamma$ , while the dashed blue line graphs relative risk aversion  $R^c$  from equation (23). The solid black line in each panel plots the model-implied average equity premium against the right axis. As is typical in standard real business cycle models (e.g., Rouwenhorst 1995), the equity premium implied by the model is small, less than about 40 basis points per year, even when  $\gamma$  and  $\chi$  are large.<sup>23</sup>

In Figure 1, the equity premium tracks relative risk aversion  $R^c$  closely and is essentially unrelated to the traditional, fixed-labor measure of risk aversion,  $\gamma$ . In the top panel,  $\gamma$  is constant at 200 while relative risk aversion  $R^c$  varies widely along with  $\chi$ , consistent with the wide variation in the equity premium. In the middle panel, the equity premium does not increase linearly along with  $\gamma$ , but rather follows a concave trajectory that matches relative risk aversion  $R^c$  closely. In the bottom panel,  $\gamma$  is constant at 200 while  $R^c$  and the equity premium increases sharply with

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<sup>23</sup>Epstein-Zin (1989) preferences solve this problem by separating risk aversion from the intertemporal elasticity of substitution—see, e.g., Rudebusch and Swanson (2012) and Swanson (2017a).



**Figure 1.** Coefficient of relative risk aversion  $R^c$  and the equity premium in a real business cycle model with labor market frictions. In each panel, one parameter is varied while the other model parameters are fixed at their benchmark values; in the top panel,  $\chi$  is varied from 0.5 to 600; in the middle panel,  $\gamma$  ranges from 0.5 to 500; and in the bottom panel,  $f(\theta)$  ranges from 0.001 to 0.7. In each panel, the equity premium tracks relative risk aversion  $R^c$  closely and is generally unrelated to the fixed-labor measure of risk aversion,  $\gamma$ . See text for details.

labor market rigidity (as the job-finding rate,  $f(\Theta)$ , approaches zero).<sup>24</sup>

The numerical example in this section thus illustrates three main points. First, the traditional, fixed-labor measure of risk aversion has essentially no relationship to the price of risky assets when households can vary their labor supply. Second, relative risk aversion  $R^c$ , defined in the present paper, is much more closely related to the equity premium. (There are good theoretical reasons to expect this to be the case; see Swanson (2017a) and the Appendix for a derivation of the relationship between the equity premium and risk aversion in the model.) And third, the difference between relative risk aversion  $R^c$  and the fixed-labor measure of risk aversion can be very large, as in the top panel of Figure 1, where relative risk aversion  $R^c$  can be arbitrarily small as  $\chi$  becomes small even while  $\gamma$  remains fixed at 200.

#### 4. Implications of Labor Market Frictions for Risk Aversion

Intuitively, labor market frictions make it more difficult for households to insure themselves from asset fluctuations by varying hours of work. Thus, a greater degree of labor market frictions should imply higher risk aversion, all else equal. That effect is evident in the bottom panel of Figure 1; in the present section, I analyze the implications of labor market frictions for risk aversion more generally.

The transition equation (2) for labor, evaluated at steady state, implies

$$sl = f(\Theta)u. \quad (37)$$

Equation (37) can be used to substitute out  $f(\Theta)u/l$  in (23) to obtain

$$R^c(a, l; \Theta) = \frac{\gamma}{1 + \frac{\gamma}{\chi} \frac{wl}{c} \frac{s + f(\Theta)}{r + s + f(\Theta)}}. \quad (38)$$

If labor is perfectly fixed, corresponding to the case  $s = f(\Theta) = 0$ , equation (38) reduces to the usual Arrow-Pratt definition,  $\gamma$ . As the ratio  $(s + f(\Theta))/(r + s + f(\Theta))$  approaches 1, equation (38) converges to the formula for risk aversion for the case where labor is perfectly flexible, reported in Swanson (2012).<sup>25</sup> Thus,  $(s + f(\Theta))/(r + s + f(\Theta))$  lies between 0 and 1 and can be

<sup>24</sup>As  $f(\Theta) \rightarrow 0$ , relative risk aversion does not approach the fixed-labor measure  $\gamma$  because the parameters  $\chi_0$  and  $\kappa$  are also changing to keep  $l + u = 0.3$  and  $v/u = 0.6$  in steady state (see Table 1 and its discussion). As a result,  $l$  and  $c$  in equation (23) both vanish as  $f(\Theta) \rightarrow 0$ , so the limit of equation (23) is not  $\gamma$ .

<sup>25</sup>Technically,  $|1 - s - f(\Theta)| < 1$  is required to solve equation (A17) forward, so  $s + f(\Theta) \in (0, 2)$  and the ratio  $(s + f(\Theta))/(r + s + f(\Theta))$  has a maximum of  $2/(r + 2)$ . However, for small enough  $r$  this ratio can be arbitrarily close to 1.

thought of as an index of labor market flexibility, with 0 corresponding to perfect rigidity and 1 to perfect flexibility. The interest rate  $r$  appears in this index because labor frictions only *delay* the household's labor adjustment, rather than preventing it, and  $r$  is related to the cost of this delay. Households that are very patient (have a low  $r$ ) view labor market frictions as less costly, because they can adjust their labor supply as desired given enough time to do so. This labor market flexibility index features prominently in the quantitative analysis of the next section.

Equation (37) can also be used to substitute out  $f(\Theta)$  in (23), giving

$$R^c(a, l; \Theta) = \frac{\gamma}{1 + \frac{\gamma}{\chi} \frac{wl}{c} \frac{s(1+(l/u))}{r + s(1+(l/u))}}. \quad (39)$$

$R^c(a, l; \Theta)$  is decreasing in  $s(1 + (l/u))$ , holding fixed the other quantities in equation (39). This suggests that greater labor market frictions (lower  $s$ ) or a recession (lower  $l/u$ ) should correspond to higher levels of risk aversion. The remainder of this section makes these two points more rigorously and investigates their quantitative importance.

The following assumption is not strictly necessary, but helps to simplify the discussion and intuition in the analysis below:

**Assumption 8.** *The elasticity  $-c_t U''(c_t)/U'(c_t) = \gamma$  for all  $c_t \in \Omega_c$ , the elasticity  $(l_t + u_t)V''(l_t + u_t)/V'(l_t + u_t) = \chi$  for all  $l_t + u_t \in \Omega_{lu}$ , and  $wl = c$ .*

Assumption 8 implies that  $U$  and  $V$  each have an isoelastic functional form, as in the numerical example above. The assumption  $wl = c$  is equivalent to  $ra + d = 0$ , from the household's flow budget constraint (3); this will be the case, for example, if there are no assets in steady state and no transfers in the model, or alternatively if lump-sum taxes offset the household's asset income.

The crucial feature of Assumption 8 is that  $\gamma$ ,  $\chi$ , and  $wl/c$  in (39) can be regarded as stable compared to  $s(1 + (l/u))$ . The intuition and basic results in the analysis below continue to hold if Assumption 8 hold only approximately rather than exactly. However, if any of  $\gamma$ ,  $\chi$ , or  $wl/c$  vary substantially, then the analytical results below may not hold and one would have to resort to numerical solutions of the model to determine the variation in (39).

#### 4.1 Risk Aversion Is Higher in More Frictional Labor Markets

Labor markets are more frictional when  $f(\Theta)$  is lower—when it is harder for an unemployed worker to find a job. The following proposition characterizes the relationship between  $f(\Theta)$  and risk aversion:

**Proposition 3.** *Let  $f_1, f_2 : \Omega_\Theta \rightarrow [0, 1]$ , and let the other parameters of the household's optimization problem be held fixed. Given Assumptions 1–8, let  $(a_1, l_1; \Theta_1)$  and  $(a_2, l_2; \Theta_2)$  denote the steady-state values of  $(a_t, l_t; \Theta_t)$  corresponding to  $f_1$  and  $f_2$ , respectively, and let  $R_1^c(a_1, l_1; \Theta_1)$  and  $R_2^c(a_2, l_2; \Theta_2)$  denote the corresponding values of risk aversion from (38). If  $f_1(\Theta_1) < f_2(\Theta_2)$ , then  $R_1^c(a_1, l_1; \Theta_1) > R_2^c(a_2, l_2; \Theta_2)$ .*

PROOF: Since  $1 + r = 1/\beta$ ,  $r$  is independent of  $f(\Theta)$ . Assumption 8 then implies  $R^c(a, l; \Theta)$  in equation (38) is decreasing in  $f(\Theta)$ .

Consistent with the intuition presented earlier, Proposition 3 shows that a more rigid labor market implies greater risk aversion. The case  $f(\Theta) = s = 0$  corresponds to complete labor market rigidity and leads to the maximal level of household risk aversion,  $\gamma$ .

Proposition 3 is interesting because it suggests that economies with more frictional labor markets should also have higher risk premia. For example, if labor markets in Europe are characterized by lower job-finding probabilities  $f(\Theta)$  than in the U.S., then Proposition 3 implies risk aversion in those countries should be higher than in the U.S. I investigate the quantitative importance of this effect below.

Proposition 3 is also interesting because it holds regardless of how the production side of the economy is specified, so long as Assumptions 1–8 for the household's problem are satisfied. The details of the production function and matching technology will generally affect the stochastic process for  $\Theta_t$  and the functional form of  $f$ , but do not affect the conclusions of the proposition.

Finally, it may be tempting to conclude from (38) that  $R^c$  is decreasing in  $s$  as well as  $f(\Theta)$ , but that is not necessarily the case. Changes in  $s$  may affect the steady-state level of  $\Theta$  and thus  $f(\Theta)$ —for example, a lower value of  $s$  tends to increase  $l/u$ , which increases  $f(\Theta)$  in standard labor market search models. Thus, the effect of  $s$  on  $R^c$  in (38) is ambiguous. The following corollary correctly characterizes the relationship between  $s$  and risk aversion:

**Corollary 2.** *Let  $s_1, s_2 \in [0, 1]$ ,  $f_1, f_2 : \Omega_\Theta \rightarrow [0, 1]$ , and let the other parameters of the household's optimization problem be held fixed. Given Assumptions 1–8, let  $(a_1, l_1; \Theta_1)$  and  $(a_2, l_2; \Theta_2)$  denote the steady-state values of  $(a_t, l_t; \Theta_t)$  corresponding to  $(s_1, f_1)$  and  $(s_2, f_2)$ , respectively, and  $R_1^c(a_1, l_1; \Theta_1)$  and  $R_2^c(a_2, l_2; \Theta_2)$  the corresponding values of risk aversion (38). If  $s_1 < s_2$  and  $f_1(\Theta_1) \leq f_2(\Theta_2)$ , then  $R_1^c(a_1, l_1; \Theta_1) > R_2^c(a_2, l_2; \Theta_2)$ . More generally, if  $s_1 + f_1(\Theta_1) < s_2 + f_2(\Theta_2)$ , then  $R_1^c(a_1, l_1; \Theta_1) > R_2^c(a_2, l_2; \Theta_2)$ .*

PROOF: Since  $1 + r = 1/\beta$ ,  $r$  is independent of  $s$  and  $f$ . Assumption 8 then implies  $R^c(a, l; \Theta)$  in equation (38) is increasing in  $s + f(\Theta)$ .

## 4.2 Risk Aversion Is Higher in Recessions

Intuitively, a recession is a period in which employment is low and unemployment is high, or  $l/u$  is low. The following proposition characterizes the relationship between  $l/u$  and risk aversion:

**Proposition 4.** *Given Assumptions 1–8 and fixed values for the parameters  $s$ ,  $\beta$ ,  $\gamma$ , and  $\chi$ ,  $R^c(a, l; \Theta)$  is decreasing in  $l/u$ .*

PROOF: Since  $1 + r = 1/\beta$ ,  $r$  is independent of  $l/u$ . Assumption 8 then implies  $R^c(a, l; \Theta)$  in equation (39) is decreasing in  $l/u$ .

Proposition 4 shows that risk aversion is higher in recessions. A lower ratio of employment to unemployment implies that it is harder for unemployed workers to find a job, because  $f(\Theta) = sl/u$ . As a result, it is more difficult for households to use the labor market to insure themselves from asset fluctuations.

Note that the source of the change in  $l/u$  in Proposition 4 is irrelevant. The ratio  $l/u$  could be lower because of a decrease in the efficiency of the matching function, a fall in firm productivity, a fall in government purchases, or a change in some other element of the economic state  $\Theta$ . Proposition 4 also holds regardless of how the production side of the economy is specified, so long as Assumptions 1–8 are satisfied. As was the case for Proposition 3, the details of the production function and matching technology will generally affect the stochastic process for  $\Theta_t$  and the functional form of  $f$ , but do not affect the conclusions of the proposition.

Although  $l/u$  is the ratio of *steady-state* employment to unemployment, low  $l/u$  is the standard way to model a recession in labor search models. Gross flows in and out of employment and unemployment are large in the U.S., so calibrated labor search models imply that employment and unemployment converge very rapidly to their steady state values, in a matter of weeks rather than quarters (e.g., Shimer, 2012; Elsby, Hobijn, and Şahin, 2013). Thus, the interpretation of low  $l/u$  in Proposition 4 as a recession is typical.

Finally, the model’s prediction that risk aversion is countercyclical is interesting because there is a great deal of empirical evidence that risk premia in financial markets are countercyclical (e.g., Fama and French, 1989; Campbell and Cochrane, 1999; Cochrane, 1999; Lettau and Ludvigson, 2010; Piazzesi and Swanson, 2008; Cooper and Priestley, 2009). Indeed, an important contribution of Campbell and Cochrane (1999) was to generate countercyclical risk aversion in an asset pricing model to better match the observed countercyclicality of risk premia in the data. Proposition 4 shows that labor market frictions provide an additional source of countercyclicality.

In Campbell and Cochrane (1999), risk aversion is high in recessions because consumption is lower than its long-run history; here, risk aversion is higher in recessions because it's harder for households to offset shocks to their portfolios. I investigate the quantitative importance of this effect below.

### 4.3 Risk Aversion Is Higher for Households that Are Less Employable

Propositions 3–4 and Corollary 2 all assumed a representative household. I now relax that assumption in order to consider the case where the economy is populated by a unit continuum of households divided into two types: a set of measure one of households of type 1, and a set of measure zero of households of type 2. The aggregate equilibrium of the economy is determined by the type 1 households, which can be thought of as “representative”. Type 2 households are assumed to be the same as those of type 1 except that it is harder for them to find a job—that is,  $f_2(\Theta) < f_1(\Theta)$  at the steady-state  $\Theta$ . (The aggregate state  $\Theta$  is the same for both household types.)

Given that the aggregate equilibrium is determined by the type 1 households, it is straightforward to check that Propositions 1–4 continue to hold in this economy for both type 1 and type 2 households. In particular, Propositions 1 and 2 give the expressions for risk aversion for both type 1 and type 2 households, and the risk aversion of the two types of households can be compared using Proposition 3. Since  $f_2(\Theta) < f_1(\Theta)$ , Proposition 3 implies that  $R_2^c(a_2, l_2; \Theta) > R_1^c(a_1, l_1; \Theta)$ .

In other words, risk aversion is higher for less employable (type 2) households. These households face greater labor market frictions than those of type 1, so it is more difficult for them to insure themselves from asset fluctuations in the labor market, and they are correspondingly more risk averse. I explore the quantitative importance of this effect below.

Some examples of households that might fit the type 2, “less employable” classification are households with less education, households nearing retirement age, and households that suffer from discrimination in the labor market. According to the theory above, these households should be more risk averse than regular, type 1 households, and hold a smaller fraction of their wealth in risky assets such as stocks.

This result provides a formal justification for why households nearing retirement should hold a greater fraction of their wealth in bonds rather than stocks: they are more risk averse.<sup>26</sup> Of

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<sup>26</sup>Gollier (2002) notes that younger households should have lower absolute risk aversion because they can distribute a given fall in wealth over more periods of consumption. That effect is not present in the infinite-horizon model of the present paper, although it is present in the numerical analysis of Bodie et al. (1992).

course, households near retirement also have a greater fraction of their wealth in financial assets and less in human capital, so retirees have a greater need to diversify their financial holdings in order to diversify their total wealth. Bodie, Merton, and Samuelson (1992) solve a calibrated life-cycle portfolio allocation model numerically and find that labor market flexibility is an important factor in the household's willingness to hold risky assets. For tractability, their analysis considers only the extreme cases of perfectly flexible and perfectly rigid labor markets, essentially the same two cases considered in Swanson (2012). The possibility that older households face greater labor market frictions than younger households is acknowledged by Bodie et al. (1992) as being potentially important, but is beyond the scope of their analysis.<sup>27</sup> The present paper shows how their analysis can be extended to the case of frictional labor markets, and provides some immediate insights into the nature and magnitude of these effects.

## 5. Empirical Evidence and Quantitative Implications

The theoretical predictions of the model above are consistent with a large and growing body of empirical evidence. First, there is substantial evidence that households vary their labor supply in response to financial shocks. Imbens, Rubin, and Sacerdote (2001) estimate that individuals who win a prize in the Massachusetts state lottery reduce their earnings by about 11 cents for every dollar won.<sup>28</sup> Cesarini, Lindqvist, Notowidigdo, and Östling (2017) corroborate and significantly extend this finding using administrative data on a large sample of Swedish lottery prize winners. They also find that individuals who win a lottery prize reduce their earnings by about 11 percent of unearned income won, and this effect rises to about 14 percent when spousal earnings are also included. Cesarini et al. (2017) also find that more than half this adjustment is along the intensive rather than extensive margin. Coronado and Perozek (2003) find that individuals who held more stocks in the late 1990s retired on average about 7 months earlier than nonstockholders. And Coile and Levine (2009) find that older individuals are about 7 percent less likely to retire in a given year after the stock market falls 30 percent. More generally, the labor supply literature

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<sup>27</sup>For example, “The ability to vary labor supply ex post induces the individual to assume greater risks in his portfolio ex ante.” (Bodie et al. 1992, p. 427); “Obviously, the opportunity to vary continuously one’s labor supply without cost is a far cry from the workings of actual labor markets. A more realistic model would allow limited flexibility in varying labor and leisure.” (ibid., p. 448); and “It is reasonable to hypothesize that, for most individuals, the degree of labor flexibility diminishes over the life cycle. For this reason, the effective human capital on which the individual can draw also declines.” (ibid., p. 446).

<sup>28</sup>That is, if the prize is \$10,000 per year for 20 years, the typical prize winner reduces his or her earnings by \$1100 per year indefinitely, according to the Imbens et al. (2001) estimates. The prizes they study are each paid annually for 20 years.

consistently estimates a significant negative effect of unearned income on hours worked (e.g., Pencavel, 1986; Killingsworth and Heckman, 1986).

There is also substantial evidence that risk premia in financial markets are higher in recessions (e.g., Fama and French, 1989; Cooper and Priestley, 2009).<sup>29</sup> A natural explanation for this finding is that investors’ risk aversion itself is higher in recessions—indeed, one of the contributions of Campbell and Cochrane (1999) was to provide a model that generates countercyclical risk aversion. Guiso, Sapienza, and Zingales (2017) provide direct evidence for countercyclical risk aversion using survey data from a large panel of Italian households before and during the 2008–09 recession. They show that these households’ risk aversion is substantially higher during the recession and, importantly, that the increase in risk aversion is independent of the household’s portfolio performance during the 2008 financial crisis. In other words, the increase in risk aversion appears to be unrelated to whether the household incurred portfolio losses or whether consumption fell below its previous (“habit”) level, presenting a challenge for habit-based models of countercyclical risk aversion.

The simple, stylized model of the present paper is consistent with all of these basic empirical facts. The remainder of the section considers the international evidence and more detailed quantitative predictions of the model.

## 5.1 International Comparison of Labor Flows and Household Portfolios

Table 2 reports international evidence on labor market flows and household portfolio allocations across risky and safe assets. Hobijn and Şahin (2007) estimate average values of  $s$  and  $f(\Theta)$  over time for a variety of OECD countries, and values for the six largest countries in their sample are reported in the first two columns of Table 2.<sup>30</sup> In the present paper, individuals may be out of the labor force, employed, or unemployed, so  $s$  in Table 2 is defined to be the sum of the monthly flow rates from employment into unemployment and into nonemployment. The job-finding rate  $f(\Theta)$  is the monthly flow rate from unemployment into employment. A striking feature of these estimates is that the job-finding rate in Continental Europe is an order of magnitude lower than in the U.S., ranging from 1.3 to 3.5 percent as opposed to more than 28 percent. By contrast,

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<sup>29</sup> See also Campbell and Cochrane (1999), Campbell (1999), Lettau and Ludvigson (2010), Piazzesi and Swanson (2008), and Cochrane and Piazzesi (2005).

<sup>30</sup> Hobijn and Şahin (2007) estimate a job separation rate for the U.S. of .006. For consistency with Shimer’s (2012) results for the U.S. reported later, I use Shimer’s estimates for the U.S. in Table 2 (for which  $s = .019$  and  $f(\Theta)$  is the same as in Hobijn and Şahin (2007)).

TABLE 2: INTERNATIONAL COMPARISON OF LABOR FLOWS AND HOUSEHOLD PORTFOLIOS

	$s$	$f(\Theta)$	percentage of households owning equities	percentage of households owning risky financial assets	share of household portfolios in currency and deposits
United States	.019	.282	48.9	49.2	12.4
United Kingdom	.009	.056	31.5	32.4	26.0
Germany	.006	.035	18.9	25.1	33.9
France	.007	.033	–	–	29.1
Spain	.012	.020	–	–	38.1
Italy	.004	.013	18.9	22.1	27.9

Labor market flows  $s$  and  $f(\Theta)$  are from Hobijn and Şahin (2007, Table 4) and Shimer (2012);  $s$  is sum of monthly flow rates from employment (E) to unemployment (U) and to nonemployment (N);  $f(\Theta)$  is monthly flow from U to E. Percentages of households owning equities (directly and indirectly) and risky financial assets are from Guiso et al. (2002, Table I.5) and are upper bounds for the U.K., Germany, and Italy (ibid., p. 7). Share of household portfolios in currency and deposits is from Ynesta (2008, Table 1). See text for details.

job separation rates are more similar across all the countries in the table.

The last three columns of Table 2 provide some statistics on portfolio allocations between risky and safe assets in each country. Data on the percentage of households owning equities is from Guiso, Haliassos, and Japelli (2002) for the year 1998 and includes indirect holdings through mutual funds and retirement accounts as well as direct equity holdings.<sup>31</sup> Data on the percentage of households holding risky financial assets (including corporate bonds as well as equities) is from the same source. The share of household portfolios in each country held in currency and deposits is from Ynesta (2008) for the year 2006.

The theme in the last three columns of Table 2 is that households in the U.S. are more likely to hold stocks and allocate a greater fraction of their portfolios to risky assets than households in the U.K. and Continental Europe. In fact, the difference between portfolio allocations in the U.S. and other countries in Table 2 are understatements because the U.K., German, and Italian data overestimate the degree of indirect stockholding in those countries, as discussed in the previous footnote. As shown by Guiso et al. (2002), these differences in portfolio holdings are robust to controlling for demographic characteristics such as household wealth or age; that is, U.S. households of a given wealth level and age are more likely to hold equities and risky financial assets than are European households of a similar wealth level and age.

<sup>31</sup>Data for France and Spain are not available in the survey. Data on indirect equity holdings for the U.K., Germany, and Italy are upper bounds because the portfolio surveys for those countries do not provide information on the type of mutual funds held, so that indirect stockholding in these countries cannot be separated from indirect holdings of other assets (Guiso et al., 2002, p. 7).

The international evidence in Table 2 is thus consistent with the theoretical points raised earlier: Labor markets in Europe are more frictional than those in the U.S., and households in Europe are more risk averse in their portfolio allocations. There are, of course, other reasons why European households might be more reluctant than U.S. households to invest in stocks; for example, stock markets in Europe are generally less liquid and more volatile than in the U.S., intermediation fees for indirect stock holding are typically higher, and the costs of acquiring financial information may be higher (Guiso and Jappelli, 2002, p. 260). But the lower liquidity and higher intermediation costs in European equity and mutual fund markets are also partly an equilibrium outcome of the lower level of household participation and activity in those markets. Thus, it is not clear that these other factors are fundamental causes of low equity market participation, as opposed to an endogenous response to European households' apparently higher risk aversion and preference for holding safer assets. Thus, it is certainly possible that this apparently higher level of risk aversion is due in part to the greater labor market rigidity faced by European households.

## 5.2 Quantitative Implications of Labor Market Frictions for Risk Aversion

In Table 3, I consider the quantitative importance of these labor market differences for risk aversion across countries. The first two rows report results for the benchmark cases of perfect labor market rigidity and perfect flexibility, essentially corresponding to the two extreme cases considered in Swanson (2012). The first two columns report the corresponding values of  $s$  and  $f(\Theta)$ , while the third column reports the ratio  $(s + f(\Theta))/(r + s + f(\Theta))$ , which lies between 0 and 1 and can be thought of as an index of labor market flexibility, as discussed earlier.<sup>32</sup> For the monthly discount rate  $r$ , Hall (2017) estimates a value of 0.7 percent per month, based on the similarity between firms' investment in "match capital" in the labor market and other types of capital like physical plant and equipment. This similarity implies that discount rates in the labor market should be relatively high and volatile, just as they are in the capital markets.

The last four columns of Table 3 report the value of relative risk aversion  $R^c$  from equation (38) for different values of the utility curvature parameters  $\gamma$  and  $\chi$ . When labor markets are perfectly rigid,  $R^c = \gamma$ , while when labor markets are perfectly flexible,  $R^c$  is smaller by a factor of three to ten for the parameterizations in the table, a dramatic difference.

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<sup>32</sup>The benchmark  $s = f(\Theta) = 1$  yields a labor market flexibility index of .997, which is slightly less than 1. One cannot generate an index value higher than this without assuming a lower value for  $r$ .

TABLE 3: QUANTITATIVE IMPORTANCE OF LABOR MARKET FRICTIONS FOR RISK AVERSION

	$s$	$f(\Theta)$	$\frac{s+f(\Theta)}{r+s+f(\Theta)}$	Relative Risk Aversion $R^c$			
				$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
				$\chi = 1.5$	$\chi = 0.5$	$\chi = 2.5$	$\chi = 10$
Theoretical labor market benchmarks:							
perfect rigidity	0	0	0	2	5	10	20
perfect flexibility	1	1	.997	0.86	0.46	2.01	6.68
International comparison:							
United States	.019	.282	.977	0.87	0.46	2.04	6.77
United Kingdom	.009	.056	.903	0.91	0.50	2.17	7.13
Germany	.006	.035	.854	0.94	0.52	2.26	7.38
France	.007	.033	.851	0.94	0.53	2.27	7.40
Spain	.012	.020	.821	0.96	0.54	2.34	7.57
Italy	.004	.013	.708	1.03	0.62	2.61	8.28

Relative risk aversion  $R^c$  from equation (38) for different values of  $\gamma$ ,  $\chi$ , and  $r$ , using estimated values of  $s$  and  $f(\Theta)$  from Hobijn and Şahin (2007) and Shimer (2012) and  $r = .007$  from Hall (2017). See notes to Table 2 and text for details.

The rest of Table 3 focuses on frictional labor markets, calibrated to the international values from Hobijn and Şahin (2007) in Table 2. For the U.S., the labor market flexibility index is .977, virtually as high as the frictionless benchmark. The values for the European country calibrations are lower, but even Italy's labor market flexibility index is about 71 percent by this metric. Intuitively, households in the model are sufficiently patient that the extra time it takes to vary employment is not extremely costly; thus, even households in the most rigid calibrations in Table 3 are able to insure themselves from asset fluctuations to some extent in the labor market.

These observations are reflected in the implied relative risk aversion coefficients  $R^c$  in the last four columns of Table 3. Each of these four columns considers a different configuration of utility curvature parameters  $\gamma$  and  $\chi$ . For all of these parameter configurations, relative risk aversion in the U.S. is very close to the frictionless benchmark and never near the traditional, fixed-labor measure. For the other countries, frictional labor markets cause risk aversion to be higher, ranging from a few percent higher in the U.K. to up to 35% higher for Italy. Thus, differences in labor market frictions can account for a quantitatively significant part of the apparent differences in risk aversion in Table 2. Note, though, that even for the most rigid countries in Table 3, the implied coefficients of relative risk aversion are still much lower than the traditional, fixed-labor benchmark.

In fact, the risk aversion estimates in Table 3 are likely to represent underestimates of the

TABLE 4: QUANTITATIVE IMPLICATIONS FOR CYCLICAL VARIATION IN RISK AVERSION

	$s$	$f(\Theta)$	$r$	$\frac{s+f(\Theta)}{r+s+f(\Theta)}$	Relative Risk Aversion $R^c$			
					$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
					$\chi = 1.5$	$\chi = 0.5$	$\chi = 2.5$	$\chi = 10$
U.S., expansion	.017	.35	.003	.995	0.86	0.46	2.01	6.70
U.S., recession	.022	.20	.011	.953	0.88	0.47	2.08	6.88
rigid labor mkt, expans.	.0036	.016	.003	.868	0.93	0.52	2.24	7.31
rigid labor mkt, recess.	.0046	.009	.011	.557	1.15	0.76	3.10	9.45

Relative risk aversion  $R^c$  from equation (38) for different values of  $\gamma$ ,  $\chi$ , and  $r$ . U.S. values of  $s$  and  $f(\Theta)$  are from Shimer (2012),  $r$  from Hall (2017). See notes to Table 3 and text for details.

true quantitative effect of labor market frictions on risk aversion. In the stylized model of the present paper, the only cost of labor market frictions is the delay in labor market adjustment, and this cost is completely characterized by the parameter  $r$ . To the extent that this delay is made more costly by borrowing constraints, depreciation of human capital, or stigma associated with longer unemployment, the cost parameter  $r$  should be taken to be higher than 0.7 percent for the purposes of the calculations in Table 3. If we take  $r = .014$  instead of .007, then the risk aversion coefficients for the U.S. are still very close to those in Table 3, but the coefficients for the more frictional countries increase by another roughly 20% (not reported in the table).

### 5.3 Quantitative Implications for Cyclical Variation in Risk Aversion

Finally, I consider whether labor market frictions can make a quantitatively significant contribution to business cycle variation in risk aversion. Recall that risk aversion may be higher in recessions simply because household consumption is lower and marginal utility is higher, as in Campbell and Cochrane (1999) and Swanson (2012, 2017a). In this section, I investigate whether labor market frictions play an additional role contributing to this variation. The job separation rate is higher and the job finding rate lower in recessions, implying that the labor market is correspondingly less flexible. A less flexible labor market implies that risk aversion should be higher, all else equal, in addition to any effects coming from the decline in consumption.

The first two rows of Table 4 report parameters calibrated to the U.S. economy in expansions and recessions. Values of  $s$  and  $f(\Theta)$  are taken from Shimer (2012), while estimates of  $r$  are from Hall (2017).<sup>33</sup> In recessions, the job separation rate increases by almost a third, the job finding

<sup>33</sup>Hall (2017) estimates a one-month discount rate of  $-0.1$  percent in expansions and 13.5 percent in recessions, and an annualized 29-month discount rate of 4.4 percent in expansions and 12.2 percent in recessions (where 29

rate falls by 40%, and the discount rate roughly triples. Nevertheless, labor markets are so flexible in the U.S. that even in the recession calibration, the index of labor market flexibility remains very high, .953. The resulting differences in risk aversion  $R^c$  between recessions and expansions in the last four columns are thus quite small.<sup>34</sup> Intuitively, labor market flows in the U.S. are simply too large to make delay very costly for households, even in recessions.

Of course, this cyclical variation tends to be larger in countries with more rigid labor markets. In the bottom panel of Table 4, I consider a calibration based on the estimates for Italy in Table 3. In particular, I take the job separation rate for the U.S. in expansions from Table 4 and scale those values down by the ratio of average job separations in Italy to that in the U.S. from Table 3. This produces a separation rate of 0.36 percent in expansions and 0.46 percent in recessions. I do the same thing for the job finding rate. Finally, I assume that the discount rate is the same as in the U.S. calibration. (I refer to this as a “rigid labor market” rather than an “Italy” calibration because I don’t have actual data on how job separation and finding rates vary over the business cycle in Italy.) In these bottom rows, the variation in risk aversion over the business cycle is much more significant, rising about 25–50% in recessions.

I conclude from this exercise that cyclical variation in labor market frictions is probably not a significant contributor to time-varying risk aversion in the U.S., but it may be a significant contributor to the cyclical variation of risk aversion in countries with more frictional labor markets, such as in Continental Europe.

## 6. Conclusions

Traditional studies of risk aversion, such as Arrow (1964), Pratt (1965), Epstein and Zin (1989), and Weil (1989), assume that household labor supply is fixed. However, this assumption ignores households’ ability to partially offset portfolio shocks by varying hours of work. As a result, fixed-labor measures of risk aversion are not representative of the household’s aversion to holding risky assets when labor supply can vary. For reasonable parameterizations, traditional, fixed-labor measures of risk aversion can overstate the household’s aversion to holding a risky asset by a factor of as much as ten, as in Table 3.

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months corresponds to the lifetime of a job in the model). In my application, the relevant period is the lifetime of the unemployment spell rather than the job, and the former is shorter than the latter. Thus, in Table 4 I use discount rates that are intermediate between Hall’s one-month and 29-month values.

<sup>34</sup>Note that these computations set consumption to its steady-state value, so the differences between expansion and recession in Table 4 are completely due to differences in labor market rigidity.

The closed-form expressions for risk aversion I derive above lie between the traditional, fixed-labor measures and the flexible-labor expressions in Swanson (2012). Traditional, fixed-labor measures of risk aversion are essentially unrelated to the equity premium in a standard real business cycle model with labor market search, while the expressions derived here match the equity premium closely. Thus, taking into account the household's labor margin is necessary for risk aversion to correspond to risk premia in the model.

According to my analysis, risk aversion is higher: 1) in countries with more frictional labor markets, such as Continental Europe, 2) in recessions, and 3) for households that are less employable, such as less-educated households, retirees, and households facing labor market discrimination. In all of these cases, it's more difficult for the household to vary hours of work in response to portfolio shocks, and so these shocks pass through to consumption to a greater extent.

These predictions are consistent with a wide variety of empirical evidence in the literature. Numerous authors find that Continental European households are more conservative in their portfolio allocations (e.g., Guiso et al., 2002), that risk premia are higher in recessions (e.g., Fama and French, 1989), and that older households invest more conservatively (e.g., Guiso et al., 2002). Many other authors find that households vary their labor supply significantly in response to asset fluctuations (e.g., Imbens et al., 2001; Cesarini et al., 2017).

Quantitatively, U.S. labor market flows are sufficiently high that household risk aversion is very close to the frictionless benchmark considered in Swanson (2012). More rigid labor markets, such as Spain and Italy, imply a higher coefficient of relative risk aversion, which I conservatively estimate to be 15–35% larger than the frictionless benchmark. Note that even these higher values are still substantially less than traditional, fixed-labor measure of risk aversion. Thus, taking the household's labor margin into account is important for understanding the household's attitudes toward holding a risky asset.

## Appendix: Proofs of Propositions and Technical Details

### Microfoundation for Household Preferences

The household consists of a unit continuum of individuals, indexed by  $i \in [0, 1]$ . At each time  $t$ , each individual either works, searches for a job, or engages in home production of nonmarket goods and services, including “leisure”. Let  $l_t$  denote the measure of household members who work in period  $t$ ,  $u_t$  the measure of household members who search, and  $h_t = 1 - l_t - u_t$  the measure of household members who engage in home production. The household’s output of nonmarket goods and services,  $c_t^n$ , is given by

$$c_t^n = H(h_t), \quad (\text{A1})$$

where  $H$  is an increasing, concave, twice-differentiable function of the measure of household members engaged in home production.

Each individual household member has a period utility function given by

$$u(c_{it}^m) + v(c_{it}^n), \quad (\text{A2})$$

where  $c_{it}^m$  and  $c_{it}^n$  denotes individual  $i$ ’s consumption at time  $t$  of market and nonmarket goods, respectively, and  $u$  and  $v$  are increasing, strictly concave, and twice-differentiable functions. For simplicity, I assume that the disutility of working, searching, and home production are equal and enter additively into (A2). Since the disutility of work then equals a constant for each individual in each period  $t$ , that constant can be normalized to zero.

Household members pool their income and consumption of home-produced goods, so that each member consumes the same amount of market and nonmarket goods in each period  $t$ . Let  $c_t$  and  $c_t^n$  denote this common level of market and nonmarket good consumption, respectively.

The household’s utility flow in each period  $t$  is given by the integral of its individual members’ utility flows, which equals

$$u(c_t) + v(H(1 - l_t - u_t)) \quad (\text{A3})$$

since each individual’s consumption of market and nonmarket goods is the same. Let  $U(c_t) \equiv u(c_t)$  and  $V(l_t + u_t) \equiv -v(H(1 - l_t - u_t))$ . Then the household’s period utility function is given by

$$U(c_t) - V(l_t + u_t), \quad (\text{A4})$$

where  $U$  is increasing, strictly concave, and twice-differentiable, and  $V$  is increasing, strictly convex, and twice-differentiable, as stated in preference specification (1).

### Proof of Proposition 1

Since  $(a_t, l_t; \Theta_t)$  is an interior point of  $X$ ,  $\mathbb{V}(a_t + \frac{\sigma \underline{\varepsilon}}{1+r_t}, l_t; \Theta_t)$  and  $\mathbb{V}(a_t + \frac{\sigma \bar{\varepsilon}}{1+r_t}, l_t; \Theta_t)$  exist for sufficiently small  $\sigma$ , and  $\mathbb{V}(a_t + \frac{\sigma \underline{\varepsilon}}{1+r_t}, l_t; \Theta_t) \leq \tilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma) \leq \mathbb{V}(a_t + \frac{\sigma \bar{\varepsilon}}{1+r_t}, l_t; \Theta_t)$ , hence  $\tilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma)$  exists. Moreover, since  $\mathbb{V}(\cdot, \cdot; \cdot)$  is continuous and increasing in its first argument, the intermediate value theorem implies there exists a unique  $-\mu(a_t, l_t; \Theta_t; \sigma) \in [\sigma \underline{\varepsilon}, \sigma \bar{\varepsilon}]$  satisfying  $\mathbb{V}(a_t - \frac{\mu}{1+r_t}, l_t; \Theta_t) = \tilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma)$ .

For a sufficiently small fee  $\mu$  in (8), the change in household welfare (6) is given to first order by

$$\frac{-\mathbb{V}_1(a_t, l_t; \Theta_t)}{1 + r_t} d\mu. \quad (\text{A5})$$

Using the envelope theorem, we can rewrite (A5) as

$$-\beta E_t \mathbb{V}_1(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}) d\mu, \quad (\text{A6})$$

where  $a_{t+1}^* \equiv (1 + r_t)a_t + w_t l_t + d_t - c_t^*$  and  $l_{t+1}^* = (1 - s)l_t + f(\Theta_t)u_t^*$ .

Turning now to the gamble in (7), note that the household’s optimal choices for consumption and unemployment in period  $t$ ,  $c_t^*$  and  $u_t^*$ , will generally depend on the size of the gamble  $\sigma$ —for example, the household may undertake precautionary saving when faced with this gamble. Thus, in this section we

write  $c_t^* \equiv c^*(a_t, l_t; \Theta_t; \sigma)$  and  $u_t^* \equiv u^*(a_t, l_t; \Theta_t; \sigma)$  to emphasize this dependence on  $\sigma$ . The household's value function, inclusive of the one-shot gamble in (7), satisfies

$$\tilde{V}(a_t, l_t; \Theta_t; \sigma) = U(c_t^*) - V(l_t + u_t^*) + \beta E_t \mathbb{V}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}). \quad (\text{A7})$$

Because (7) describes a one-shot gamble in period  $t$ , it affects assets  $a_{t+1}^*$  and  $l_{t+1}^*$  in period  $t+1$  but otherwise does not affect the household's optimization problem from period  $t+1$  onward; as a result, the household's value-to-go at time  $t+1$  is just  $\mathbb{V}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1})$ , which does not depend on  $\sigma$  except through  $a_{t+1}^*$  and  $l_{t+1}^*$ .

Differentiating (A7) with respect to  $\sigma$ , the first-order effect of the gamble on household welfare is

$$\left[ U' \frac{\partial c_t^*}{\partial \sigma} - V' \frac{\partial u_t^*}{\partial \sigma} + \beta E_t \mathbb{V}_1 \cdot \left( -\frac{\partial c_t^*}{\partial \sigma} + \varepsilon_{t+1} \right) + \beta E_t \mathbb{V}_2 f(\Theta_t) \frac{\partial u_t^*}{\partial \sigma} \right] d\sigma, \quad (\text{A8})$$

where the arguments of  $U'$ ,  $V'$ ,  $\mathbb{V}_1$ , and  $\mathbb{V}_2$  are suppressed to reduce notation. Optimality of  $c_t^*$  and  $u_t^*$  implies that the terms involving  $\partial c_t^*/\partial \sigma$  and  $\partial u_t^*/\partial \sigma$  in (A8) cancel, as in the usual envelope theorem (these derivatives vanish at  $\sigma = 0$  anyway, for the reasons discussed below). Moreover,  $E_t \mathbb{V}_1(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}) \varepsilon_{t+1} = 0$  because  $\varepsilon_{t+1}$  is independent of  $\Theta_{t+1}$ ,  $l_{t+1}^*$ , and  $a_{t+1}^*$ , evaluating the latter two at  $\sigma = 0$ . Thus, the first-order cost of the gamble is zero, as in Arrow (1964) and Pratt (1965).

To second order, the effect of the gamble on household welfare is

$$\begin{aligned} & \left[ U'' \left( \frac{\partial c_t^*}{\partial \sigma} \right)^2 - V'' \left( \frac{\partial u_t^*}{\partial \sigma} \right)^2 + U' \frac{\partial^2 c_t^*}{\partial \sigma^2} - V' \frac{\partial^2 u_t^*}{\partial \sigma^2} \right. \\ & + \beta E_t \mathbb{V}_{11} \cdot \left( -\frac{\partial c_t^*}{\partial \sigma} + \varepsilon_{t+1} \right)^2 + \beta E_t \mathbb{V}_{12} \cdot \left( -\frac{\partial c_t^*}{\partial \sigma} + \varepsilon_{t+1} \right) f(\Theta_t) \frac{\partial u_t^*}{\partial \sigma} + \beta E_t \mathbb{V}_1 \cdot \left( -\frac{\partial^2 c_t^*}{\partial \sigma^2} \right) \\ & \left. + \beta E_t \mathbb{V}_{22} f(\Theta_t)^2 \left( \frac{\partial u_t^*}{\partial \sigma} \right)^2 + \beta E_t \mathbb{V}_{12} \cdot \left( -\frac{\partial c_t^*}{\partial \sigma} + \varepsilon_{t+1} \right) f(\Theta_t) \frac{\partial u_t^*}{\partial \sigma} + \beta E_t \mathbb{V}_2 f(\Theta_t) \frac{\partial^2 u_t^*}{\partial \sigma^2} \right] \frac{d\sigma^2}{2}. \end{aligned} \quad (\text{A9})$$

The terms involving  $\partial^2 c_t^*/\partial \sigma^2$  and  $\partial^2 u_t^*/\partial \sigma^2$  cancel due to the optimality of  $c_t^*$  and  $u_t^*$ . The derivatives  $\partial c_t^*/\partial \sigma$  and  $\partial u_t^*/\partial \sigma$  also vanish at  $\sigma = 0$  (there are two ways to see this: first, the linearized version of the model is certainty equivalent; alternatively, if  $\varepsilon$  is symmetric, the gamble in (7) is isomorphic for positive and negative  $\sigma$ , hence  $c^*$  and  $u^*$  must be symmetric about  $\sigma = 0$ , implying the derivatives vanish). Thus, for infinitesimal gambles, (A9) simplifies to

$$\beta E_t \mathbb{V}_{11}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}) \varepsilon_{t+1}^2 \frac{d\sigma^2}{2}. \quad (\text{A10})$$

Finally,  $\varepsilon_{t+1}$  is independent of  $\Theta_{t+1}$ ,  $l_{t+1}^*$ , and  $a_{t+1}^*$ , evaluating the latter two at  $\sigma = 0$ . Since  $\varepsilon_{t+1}$  has unit variance, (A10) reduces to

$$\beta E_t \mathbb{V}_{11}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}) \frac{d\sigma^2}{2}. \quad (\text{A11})$$

Equating (A6) to (A11) allows us to solve for  $d\mu$  as a function of  $d\sigma^2$ . Thus,  $\lim_{\sigma \rightarrow 0} 2\mu(a_t, l_t; \Theta_t; \sigma)/\sigma^2$  exists and is given by

$$\frac{-E_t \mathbb{V}_{11}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1})}{E_t \mathbb{V}_1(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1})}. \quad (\text{A12})$$

To evaluate (A12) at the nonstochastic steady state, set  $a_{t+1} = a$ ,  $l_{t+1} = l$ , and  $\Theta_{t+1} = \Theta$  to get

$$\frac{-\mathbb{V}_{11}(a, l; \Theta)}{\mathbb{V}_1(a, l; \Theta)}. \quad (\text{A13})$$

### Proof of Lemma 1

The computation in the lemma is performed in two steps, using the household's Euler equation for unemployment,

$$\frac{V'(l_\tau^* + u_\tau^*)}{f(\Theta_\tau)} = \beta E_\tau \left[ w_{\tau+1} U'(c_{\tau+1}^*) - V'(l_{\tau+1}^* + u_{\tau+1}^*) + (1-s) \frac{V'(l_{\tau+1}^* + u_{\tau+1}^*)}{f(\Theta_{\tau+1})} \right], \quad (\text{A14})$$

and the transition equation (2) for labor. Equation (A14) is derived in the same way as the consumption Euler equation and is standard in the labor search literature. The left-hand side of (A14) represents the household's marginal cost of finding a job, while the right-hand side is the discounted marginal benefit of having one more employed worker, given by the wage times the marginal utility of consumption less the marginal disutility of work, plus the job retention rate  $(1-s)$  times the marginal benefit of not having to search for a job next period.

Equation (A14) holds at each date  $\tau \geq t$  for any initial asset stock  $a_t$ . Differentiating (A14) with respect to  $a_t$  and taking time- $t$  expectations yields

$$E_t \frac{V''(l_\tau^* + u_\tau^*)}{f(\Theta_\tau)} \left( \frac{\partial l_\tau^*}{\partial a_t} + \frac{\partial u_\tau^*}{\partial a_t} \right) = \beta E_t \left[ w_{\tau+1} U''(c_{\tau+1}^*) \frac{\partial c_{\tau+1}^*}{\partial a_t} + V''(l_{\tau+1}^* + u_{\tau+1}^*) \frac{1-s-f(\Theta_{\tau+1})}{f(\Theta_{\tau+1})} \left( \frac{\partial l_{\tau+1}^*}{\partial a_t} + \frac{\partial u_{\tau+1}^*}{\partial a_t} \right) \right]. \quad (\text{A15})$$

Evaluating (A15) at steady state gives

$$E_t \left( \frac{\partial l_\tau^*}{\partial a_t} + \frac{\partial u_\tau^*}{\partial a_t} \right) = \beta w \frac{U''(c)f(\Theta)}{V''(l+u)} E_t \frac{\partial c_{\tau+1}^*}{\partial a_t} + \beta(1-s-f(\Theta)) E_t \left( \frac{\partial l_{\tau+1}^*}{\partial a_t} + \frac{\partial u_{\tau+1}^*}{\partial a_t} \right). \quad (\text{A16})$$

Substituting for  $\partial c_{\tau+1}^*/\partial a_t$  from (15), and for  $w$  from (A14) evaluated at steady state, this becomes

$$E_t \left( \frac{\partial l_\tau^*}{\partial a_t} + \frac{\partial u_\tau^*}{\partial a_t} \right) = -\frac{\gamma}{\chi} \frac{l+u}{c} [1-\beta(1-s-f(\Theta))] \frac{\partial c_t^*}{\partial a_t} + \beta(1-s-f(\Theta)) E_t \left( \frac{\partial l_{\tau+1}^*}{\partial a_t} + \frac{\partial u_{\tau+1}^*}{\partial a_t} \right), \quad (\text{A17})$$

where  $\gamma \equiv -cU''(c)/U'(c)$  and  $\chi \equiv (l+u)V''(l+u)/V'(l+u)$ . Since  $|1-s-f(\Theta)| < 1$ , equation (A17) can be solved forward to yield

$$E_t \left( \frac{\partial l_\tau^*}{\partial a_t} + \frac{\partial u_\tau^*}{\partial a_t} \right) = -\frac{\gamma}{\chi} \frac{l+u}{c} \frac{\partial c_t^*}{\partial a_t}. \quad (\text{A18})$$

In response to a change in assets, household leisure  $(1-l_t-u_t)$  and consumption move in the same direction. (Intuitively, it is natural to think of both consumption and leisure as normal goods, so that  $\partial c_t^*/\partial a_t > 0$  and  $\partial(l_t^* + u_t^*)/\partial a_t < 0$ , although this sign restriction is not required below.)

The transition equation (2) for labor implies

$$E_t \frac{\partial l_{t+k}^*}{\partial a_t} = (1-s) E_t \frac{\partial l_{t+k-1}^*}{\partial a_t} + f(\Theta_{t+k-1}) E_t \frac{\partial u_{t+k-1}^*}{\partial a_t}. \quad (\text{A19})$$

Evaluating (A19) at steady state and applying (A18) gives

$$E_t \frac{\partial l_{t+k}^*}{\partial a_t} = (1-s-f(\Theta)) E_t \frac{\partial l_{t+k-1}^*}{\partial a_t} - \frac{\gamma}{\chi} \frac{l+u}{c} f(\Theta) \frac{\partial c_t^*}{\partial a_t}. \quad (\text{A20})$$

Solving (A20) backward to the initial condition  $\partial l_t/\partial a_t = 0$  gives

$$E_t \frac{\partial l_{t+k}^*}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l+u}{c} \frac{f(\Theta)}{s+f(\Theta)} [1-(1-s-f(\Theta))^k] \frac{\partial c_t^*}{\partial a_t}. \quad (\text{A21})$$

### Relationship between the Stochastic Discount Factor, Risk Premia, and Risk Aversion

The household's aversion to gambles over asset values depends on its ability to offset the outcome of those gambles with changes in hours worked. Here, the analysis is extended to show the relationship between risk aversion and risk premia in the Lucas-Breeden stochastic discounting framework. Risk premia in this framework are closely related to the definition of risk aversion in the present paper, and are generally unrelated to traditional, fixed-labor measures of risk aversion.

Let  $p_t^i$  denote the ex-dividend time- $t$  price of an asset  $i$  that pays stochastic dividend  $d_t^i$  each period. In equilibrium,  $p_t^i$  satisfies

$$p_t^i = E_t m_{t+1} (d_{t+1}^i + p_{t+1}^i), \quad (\text{A22})$$

where  $m_{t+1} \equiv \beta U'(c_{t+1})/U'(c_t)$  denotes the household's stochastic discount factor.

Let  $1 + r_{t+1}^i$  denote the realized gross return on the asset,

$$1 + r_{t+1}^i \equiv \frac{d_{t+1}^i + p_{t+1}^i}{p_t^i}, \quad (\text{A23})$$

and define the risk premium on the asset,  $\psi_t^i$ , to be its expected excess return,

$$\psi_t^i \equiv E_t r_{t+1}^i - r_{t+1}^f, \quad (\text{A24})$$

where  $1 + r_{t+1}^f \equiv 1/E_t m_{t+1}$  denotes the risk-free rate. Then

$$\begin{aligned} \psi_t^i &= \frac{E_t m_{t+1} E_t (d_{t+1}^i + p_{t+1}^i) - E_t m_{t+1} (d_{t+1}^i + p_{t+1}^i)}{p_t^i E_t m_{t+1}} \\ &= \frac{-\text{Cov}_t(m_{t+1}, r_{t+1}^i)}{E_t m_{t+1}}, \end{aligned} \quad (\text{A25})$$

where  $\text{Cov}_t$  denotes the covariance conditional on information at time  $t$ .

Conditional on information at time  $t$ , the household's stochastic discount factor can be written to first order as

$$dm_{t+1} = -\frac{\beta\gamma}{c} dc_{t+1}^*, \quad (\text{A26})$$

where  $\gamma \equiv cU''/U'$ . The first-order change in consumption,  $dc_{t+1}^*$ , in turn can be computed from the household's Euler equation and budget constraint, where it is now recognized that  $w_t$ ,  $r_t$ ,  $d_t$ , and  $\Theta_t$  may vary in response to economic shocks. This computation is summarized in the following lemma:

**Lemma A1.** *Let  $\varphi \equiv \frac{s + f(\Theta)}{r + s + f(\Theta)}$ , and let  $\lambda \equiv \frac{\gamma l}{\chi c}$ , the long-run sensitivity of labor relative to consumption in response to a change in assets. To first order, evaluated at steady state,*

$$\begin{aligned} dc_{t+1}^* &= \frac{r}{1 + w\lambda\varphi} \left[ da_{t+1} + \frac{w}{r + s + f(\Theta)} dl_{t+1} \right. \\ &\quad \left. + E_{t+1} \sum_{k=1}^{\infty} \frac{1}{(1+r)^k} \left( l dw_{t+k} + dd_{t+k} + adr_{t+k} + \frac{w}{r + s + f(\Theta)} udf_{t+k} \right) \right] \\ &\quad - \frac{c}{\gamma} E_{t+1} \sum_{k=1}^{\infty} \frac{1}{(1+r)^k} \left( \beta dr_{t+k+1} - \frac{rw\lambda\varphi}{1 + w\lambda\varphi} d\omega_{t+k} \right), \end{aligned} \quad (\text{A27})$$

where  $df_{t+k}$  denotes the first-order change in the job-finding rate  $f(\Theta_{t+k})$  and  $d\omega_{t+k}$  denotes the first-order change in the return to searching for a job at time  $t+k$ , defined in the Appendix.

PROOF: See below.

Note that if  $w$ ,  $r$ ,  $d$ , and  $f(\Theta)$  do not change, as in the Arrow-Pratt gamble for a single household in Section 3, then equation (A27) reduces to (25), where  $dl_t = 0$  in that example because labor at time  $t$  is given. More generally, (A27) includes the effects of changes in  $w$ ,  $r$ ,  $d$ , and  $f(\Theta)$  on consumption. The term in square brackets in (A27) describes the change in household wealth—including nonfinancial wealth—and thus the first line of (A27) describes the wealth effect on consumption. The last line of (A27) describes the substitution effect: changes in consumption due to changes in current and future wages and interest rates. The ratio  $\frac{w}{r+s+f(\Theta)}$  corresponds to the present discounted value of wages from an increase in labor at time  $t$ , over the lifetime of the job. The term  $udf_{t+k}$ , which represents the change in the number of jobs found each period, is multiplied by the same present value.

Let  $d\hat{A}_{t+1}$  denote the quantity inside the square brackets in (A27), representing the change in household wealth, and let  $d\Phi_{t+1} \equiv E_{t+1} \sum_{k=1}^{\infty} (1+r)^{-k} (\beta dr_{t+k+1} - \frac{r\lambda\varphi}{1+w\lambda\varphi} d\omega_{t+k})$ , the intertemporal substitution term from (A27). Then the stochastic discount factor and risk premium can be written as follows:

**Proposition A1.** *To first order, evaluated at steady state,*

$$dm_{t+1} = -R^a(a, l; \Theta) \beta d\hat{A}_{t+1} + \beta d\Phi_{t+1}. \quad (\text{A28})$$

*To second order, evaluated at steady state,*

$$\psi_t^i = R^a(a, l; \Theta) \text{Cov}_t(dr_{t+1}^i, d\hat{A}_{t+1}) - \text{Cov}_t(dr_{t+1}^i, d\Phi_{t+1}). \quad (\text{A29})$$

PROOF: Note first that  $R^a(a, l; \Theta) = \frac{\gamma}{c} \frac{r}{1+w\lambda\varphi}$ . Substituting (A27) into (A26) yields (A28). Substituting (A28) into (A25) yields (A29). Finally,  $\text{Cov}(dx, dy)$  is accurate to second order when  $dx$  and  $dy$  are accurate to first order.

Equation (A29) characterizes the relationship between risk aversion and the risk premium on the asset. The first term in (A29) shows that  $\psi_t^i$  increases locally linearly with  $R^a$ , by an amount that depends on the covariance between the asset return and the household's wealth, including nonfinancial wealth. This link between risk premia and risk aversion should not be too surprising: Propositions 1–2 described the risk premium for extremely simple, idiosyncratic gambles over household wealth, while Proposition A1 shows that the same coefficient also appears in the household's aversion to more general financial market gambles that may be correlated with aggregate variables such as interest rates, wages, and transfers.

The second term in (A29) corresponds to Merton's (1973) "changes in investment opportunities" in the ICAPM framework. Even if  $R^a = 0$ —that is, even if households are risk-neutral in a cross-sectional or CAPM sense— $\psi_t^i$  can be nonzero. This is because even a risk-neutral household can benefit from an asset that pays off well when the price of the household's total consumption bundle is low. An asset that pays off well when current and future wages are low (and hence leisure is cheap) or current and future interest rates are high (and hence future consumption is cheap) is preferable to an asset that pays off poorly in those situations. Even a risk-neutral household would be willing to pay a premium for such an asset—implying a lower  $\psi_t^i$ —and this effect is captured by the second term in (A29).

The fact that households in the present paper face a consumption-leisure tradeoff as well as a current- vs.-future consumption tradeoff implies that the second term in (A29) is more general than just changes in the household's investment opportunities. Indeed, the second term in (A29) is better described as being due to "changes in purchasing opportunities." The decomposition in (A29) also suggests that  $\psi_t^i$  is more accurately described as an "expected excess return" rather than a "risk premium" because only the first term in (A29) represents compensation to the household for bearing risk; the second term is not compensation for risk but rather reflects changes in the household's purchasing opportunities over time.

Finally, the decomposition (A29) can be written in terms of relative rather than absolute risk aversion using Definition 2:

**Corollary A1.** *In terms of relative risk aversion, the risk premium in (A29) can be written as:*

$$\psi_t^i = R^c(a, l; \Theta) \text{Cov}_t \left( dr_{t+1}^i, \frac{d\hat{A}_{t+1}}{A} \right) - \text{Cov}_t(dr_{t+1}^i, d\Phi_{t+1}) \quad (\text{A30})$$

where  $A$  is as in Definition 2.

**Proof of Lemma A1**

Differentiating the household's Euler equation (13) and evaluating at steady state yields

$$U''(c)(dc_t^* - E_t dc_{t+1}^*) = \beta U'(c) E_t dr_{t+1}. \quad (\text{A31})$$

It follows that, for each  $k = 1, 2, \dots$ ,

$$E_t dc_{t+k}^* = dc_t^* + \frac{\beta c}{\gamma} E_t \sum_{i=1}^k dr_{t+i}. \quad (\text{A32})$$

Combining (3)–(4), differentiating, and evaluating at steady state yields

$$E_t \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} (dc_{t+k}^* - w dl_{t+k}^* - l dw_{t+k} - dd_{t+k} - a dr_{t+k}) = (1+r) da_t. \quad (\text{A33})$$

Differentiating the household's unemployment Euler equation (17) and evaluating at steady state yields

$$\begin{aligned} \frac{V''(l+u)}{f(\Theta)} (dl_{\tau}^* + du_{\tau}^*) - \frac{V'(l+u)}{f(\Theta)^2} df_{\tau} &= \beta E_{\tau} \left[ w U''(c) dc_{\tau+1}^* + U'(c) dw_{\tau+1} \right. \\ &\left. + V''(l+u) \frac{1-s-f(\Theta)}{f(\Theta)} (dl_{\tau+1}^* + du_{\tau+1}^*) - V'(l+u) \frac{1-s}{f(\Theta)^2} df_{\tau+1} \right]. \end{aligned} \quad (\text{A34})$$

Rearranging (A34) gives

$$\begin{aligned} dl_{\tau}^* + du_{\tau}^* &= \beta(1-s-f(\Theta)) E_{\tau} (dl_{\tau+1}^* + du_{\tau+1}^*) - \frac{\gamma l+u}{\chi c} (1-\beta(1-s-f(\Theta))) E_{\tau} dc_{\tau+1}^* \\ &+ \frac{l+u}{\chi w} (1-\beta(1-s-f(\Theta))) E_{\tau} dw_{\tau+1} + \frac{l+u}{\chi f(\Theta)} df_{\tau} - \beta \frac{l+u}{\chi} \frac{1-s}{f(\Theta)} E_{\tau} df_{\tau+1}, \end{aligned} \quad (\text{A35})$$

which, assuming  $|s+f(\Theta)| < 1$ , can be solved forward to yield

$$\begin{aligned} dl_{\tau}^* + du_{\tau}^* &= -\frac{\gamma l+u}{\chi c} dc_{\tau}^* - \beta \frac{l+u}{\chi} E_{\tau} \sum_{k=1}^{\infty} \beta^{k-1} (1-s-f(\Theta))^{k-1} dr_{\tau+k} \\ &+ \frac{l+u}{\chi} (1-\beta(1-s-f(\Theta))) E_{\tau} \sum_{k=1}^{\infty} \beta^{k-1} (1-s-f(\Theta))^{k-1} dw_{\tau+k} \\ &+ \frac{l+u}{\chi f(\Theta)} df_{\tau} - \beta \frac{l+u}{\chi} E_{\tau} \sum_{k=1}^{\infty} \beta^{k-1} (1-s-f(\Theta))^{k-1} df_{\tau+k}. \end{aligned} \quad (\text{A36})$$

Note that, when current and future  $r$ ,  $w$ , and  $f(\Theta)$  do not change, (A36) reduces to (21). Define

$$\begin{aligned} d\omega_{\tau} &\equiv -\beta E_{\tau} \sum_{k=1}^{\infty} \beta^{k-1} (1-s-f(\Theta))^{k-1} dr_{\tau+k} + (1-\beta(1-s-f(\Theta))) E_{\tau} \sum_{k=1}^{\infty} \beta^{k-1} (1-s-f(\Theta))^{k-1} dw_{\tau+k} \\ &+ \frac{1}{f(\Theta)} df_{\tau} - \beta E_{\tau} \sum_{k=1}^{\infty} \beta^{k-1} (1-s-f(\Theta))^{k-1} df_{\tau+k}. \end{aligned} \quad (\text{A37})$$

Then

$$dl_{\tau}^* + du_{\tau}^* = -\frac{\gamma}{\chi} \frac{l+u}{c} dc_{\tau}^* - \frac{l+u}{\chi} d\omega_{\tau}, \quad (\text{A38})$$

so that (A38) corresponds to a standard intratemporal optimality condition between  $l+u$  and  $c$ , and  $d\omega_{\tau}$  represents the change in the present discounted value of the gains from searching for a job, analogous to  $dw_{\tau}$  in a frictionless model.

Differentiating the transition equation (2) for labor and evaluating at steady state yields

$$dl_{t+k}^* = (1-s-f(\Theta)) dl_{t+k-1}^* + f(\Theta_{t+k-1}) (dl_{t+k-1}^* + du_{t+k-1}^*) + udf_{t+k-1}. \quad (\text{A39})$$

Solving backward to time  $t$  gives

$$dl_{t+k}^* = (1-s-f(\Theta))^k dl_t + \sum_{i=1}^{\infty} (1-s-f(\Theta))^{k-1-i} [f(\Theta)(dl_{t+i}^* + du_{t+i}^*) + udf_{t+i}]. \quad (\text{A40})$$

Substituting (A38) into (A40) and taking expectations gives

$$\begin{aligned} E_t dl_{t+k}^* &= (1-s-f(\Theta))^k dl_t - \frac{\gamma}{\chi} \frac{l+u}{c} f(\Theta) \frac{1-(1-s-f(\Theta))^k}{s+f(\Theta)} dc_t^* \\ &\quad + \frac{l+u}{\chi} f(\Theta) E_t \sum_{i=0}^{k-1} (1-s-f(\Theta))^{k-1-i} d\omega_{t+i} + u E_t \sum_{i=0}^{k-1} (1-s-f(\Theta))^{k-1-i} df_{t+i} \\ &\quad - \beta \frac{l+u}{\chi} \frac{f(\Theta)}{s+f(\Theta)} E_t \sum_{i=1}^{k-1} [1-(1-s-f(\Theta))^{k-i}] dr_{t+i}. \end{aligned} \quad (\text{A41})$$

Inserting (A41) into the budget constraint (A33) gives

$$\begin{aligned} \frac{1+r}{r} \left[ 1 + \frac{\gamma}{\chi} \frac{wl}{c} \frac{s+f(\Theta)}{r+s+f(\Theta)} \right] dc_t^* - \frac{1+r}{r+s+f(\Theta)} w dl_t - \frac{1}{r+s+f(\Theta)} E_t \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} w u df_{t+k} \\ - \frac{1}{\chi} \frac{s+f(\Theta)}{r+s+f(\Theta)} E_t \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} l d\omega_{t+k} + \frac{1+r}{r} \frac{\beta c}{\gamma} \left[ 1 + \frac{\gamma}{\chi} \frac{wl}{c} \frac{s+f(\Theta)}{r+s+f(\Theta)} \right] E_t \sum_{k=1}^{\infty} \frac{1}{(1+r)^k} dr_{t+k} \\ - E_t \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} [l d\omega_{t+k} + dd_{t+k} + adr_{t+k}] = (1+r) da_t. \end{aligned} \quad (\text{A42})$$

Solving for  $dc_t^*$ , using the definitions of  $\lambda$  and  $\varphi$ , yields

$$\begin{aligned} dc_t^* &= \frac{r}{1+r} \frac{1}{1+w\lambda\varphi} \left[ (1+r) da_t + (1+r) \frac{w}{r+s+f(\Theta)} dl_t \right. \\ &\quad \left. + E_t \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} \left( l d\omega_{t+k} + dd_{t+k} + adr_{t+k} + \frac{wu}{r+s+f(\Theta)} df_{t+k} \right) \right] \\ &\quad - \frac{\beta c}{\gamma} E_t \sum_{k=1}^{\infty} \frac{1}{(1+r)^k} dr_{t+k} + \frac{r}{1+r} \frac{c}{\gamma} \frac{w\lambda\varphi}{1+w\lambda\varphi} E_t \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} d\omega_{t+k}. \end{aligned} \quad (\text{A43})$$

### Expressions for Risk Aversion with Balanced Growth

The results in the main text carry through essentially unchanged to the case of balanced growth. The corresponding expressions are collected here in Lemma A2, Proposition A2, and Corollary A2.

Along a balanced growth path,  $x \in \{l, r, u\}$  satisfies  $x_{t+k} = x_t$  for  $k = 1, 2, \dots$ , and I drop the time subscript to denote the constant value. For  $x \in \{a, c, w, d\}$ , we have  $x_{t+k} = G^k x_t$  for  $k = 1, 2, \dots$ , for

some  $G \in (0, 1+r)$ , and we let  $x_t^{bg}$  denote the balanced growth path value. I denote the balanced growth path value of  $\Theta_t$  by  $\Theta_t^{bg}$ , although the elements of  $\Theta$  may grow at different constant rates over time (or remain constant). The job-finding rate  $f(\Theta_t^{bg})$  must be constant along the balanced growth path because  $f(\Theta_t^{bg}) = sl/u$ , and I write this value as  $f(\Theta)$ . Additional details regarding balanced growth are provided in King, Plosser, and Rebelo (1988, 2002).

**Lemma A2.** *Given Assumptions 1–6 and 7', along the balanced growth path,  $\partial c_{t+k}^*/\partial a_t = G^k \partial c_t^*/\partial a_t$ ,  $k = 1, 2, \dots$ , and*

$$\frac{\partial c_t^*}{\partial a_t} = \frac{1+r-G}{1 + \frac{\gamma}{\chi} \frac{w_t^{bg} l}{c_t^{bg}} \frac{s+f(\Theta)}{(\frac{1+r}{G}-1) + s + f(\Theta)}}. \quad (\text{A44})$$

PROOF: The household's Euler equation (13), evaluated along the (nonstochastic) balanced growth path, implies

$$U''(c_t^{bg}) = \beta(1+r)U''(c_{t+1}^{bg}) = \beta(1+r)U''(Gc_t^{bg}). \quad (\text{A45})$$

As in King, Plosser, and Rebelo (2002), assume that preferences  $U$  are consistent with balanced growth for all initial asset stocks and wages in a neighborhood of  $a_t^{bg}$  and  $w_t^{bg}$ , and hence for all initial values of  $(c_t, l_t)$  in a neighborhood of  $(c_t^{bg}, l)$ . Thus, (A45) can be differentiated to yield

$$U''(c_t^{bg}) = \beta(1+r)G U''(Gc_t^{bg}). \quad (\text{A46})$$

Differentiating (A45) with respect to  $a_t$  yields

$$U''(c_t^{bg}) \frac{\partial c_t^*}{\partial a_t} = \beta(1+r)U''(c_{t+1}^{bg}) \frac{\partial c_{t+1}^*}{\partial a_t}. \quad (\text{A47})$$

Solving for  $\partial c_{t+1}^*/\partial a_t$  and using (A46) yields  $\partial c_{t+1}^*/\partial a_t = G \partial c_t^*/\partial a_t$ .

Next, use the household's budget constraint (1)–(2) and the above to solve for  $\partial c_t^*/\partial a_t$ , following along the lines of (16)–(25).

The larger is  $G$ , the smaller is  $\partial c_t^*/\partial a_t$ , since the household chooses to absorb a greater fraction of asset shocks in future periods.

**Proposition A2.** *Given Assumptions 1–6 and 7', absolute risk aversion satisfies*

$$R^a(a_t^{bg}, l; \Theta_t^{bg}) = \frac{-\mathbb{V}_{11}(a_{t+1}^{bg}, l_{t+1}^{bg}; \Theta_{t+1}^{bg})}{\mathbb{V}_1(a_{t+1}^{bg}, l_{t+1}^{bg}; \Theta_{t+1}^{bg})} \quad (\text{A48})$$

and

$$R^a(a_t^{bg}, l; \Theta_t^{bg}) = \frac{-U''(c_t^{bg})}{U'(c_t^{bg})} \frac{\frac{1+r}{G} - 1}{1 + \frac{\gamma}{\chi} \frac{w_t^{bg} l}{c_t^{bg}} \frac{s+f(\Theta)}{(\frac{1+r}{G}-1) + s + f(\Theta)}}. \quad (\text{A49})$$

PROOF: Proposition 1 implies (A48). Assumptions 1–6 imply (11)–(12). Substituting (11)–(12) and Lemma A2 into (A48) gives

$$R^a(a_t^{bg}, l; \Theta_t^{bg}) = \frac{-U''(c_{t+1}^{bg})}{U'(c_{t+1}^{bg})} \frac{1+r-G}{1 + \frac{\gamma}{\chi} \frac{w_t^{bg} l}{c_t^{bg}} \frac{G(s+f(\Theta))}{(1+r-G) + G(s+f(\Theta))}}. \quad (\text{A50})$$

Applying the Euler equation (13) and (A45) completes the proof.

Note that (A50) agrees with Proposition 2 when  $G = 1$ . The larger is  $G$ , the smaller is  $R^a$ , since larger  $G$  implies greater household wealth and ability to absorb asset shocks.

**Corollary A2.** *Given Assumptions 1–6 and 7', relative risk aversion satisfies*

$$R^c(a_t^{bg}, l; \Theta_t^{bg}) = \frac{\gamma}{1 + \frac{\gamma}{\chi} \frac{w_t^{bg} l}{c_t^{bg}} \frac{s + f(\Theta)}{(\frac{1+r}{G} - 1) + s + f(\Theta)}}. \quad (\text{A51})$$

PROOF: As in Definition 2, define wealth  $A_t^{bg}$  in beginning- rather than end-of-period- $t$  units; this requires multiplying by  $((1+r)/G)^{-1}$  rather than just  $(1+r)^{-1}$ . Then the present discounted value of consumption along the balanced growth path is given by  $A_t^{bg} = c_t^{bg} / (\frac{1+r}{G} - 1)$ . Substituting these into Proposition A2 completes the proof.

Thus, the expressions for relative risk aversion are largely unchanged by balanced growth. The labor market flexibility index takes the form  $\frac{s + f(\Theta)}{(\frac{1+r}{G} - 1) + s + f(\Theta)}$  rather than  $\frac{s + f(\Theta)}{r + s + f(\Theta)}$ , which has the effect of decreasing the importance of  $r$  and amplifying the importance of  $s + f(\Theta)$  in the index, but other than that the expression is unchanged.

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