Optimal Time-Consistent Monetary Policy in the New Keynesian Model with Repeated Simultaneous Play

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Summary

- There are two definitions of “discretion” in the literature
- These definitions differ in terms of within-period timing of play
- Within-period timing makes a *huge* difference
- In the New Keynesian model with repeated Stackelberg play, there are multiple equilibria (King-Wolman, 2004)
- In the New Keynesian model with repeated simultaneous play, there is a unique equilibrium (this paper)
- Empirical relevance: Will the 1970s repeat itself?

Background and Motivation

Time-consistent (discretionary) policy: Kydland and Prescott (1977)

There are multiple equilibria under discretion:

- Barro and Gordon (1983)
- Chari, Christiano, Eichenbaum (1998)

Critiques of the Barro-Gordon/CEE result:

- enormous number, range of equilibria make theory impossible to test or reject
- equilibria require fantastic sophistication, coordination across continuum of atomistic agents

Background and Motivation

Literature has thus changed focus to *Markov perfect equilibria*:

- Albanesi, Chari, Christiano (2003)
- King and Wolman (2004)

King and Wolman (2004):

- standard New Keynesian model
- assume repeated Stackelberg within-period play
- there are two Markov perfect equilibria

But recall LQ literature:

- Svensson-Woodford (2003, 2004), Woodford (2003)
- Pearlman (1994)
- assume repeated simultaneous within-period play

Comparison: Fiscal Policy

Cohen and Michel (1988), Ortigueira (2005):

- two definitions of discretion in the tax literature
- Brock-Turnovsky (1980), Judd (1998): repeated simultaneous
- Klein, Krusell, Rios-Rull (2004): repeated Stackelberg
- different timing assumption lead to different equilibria, welfare

In this paper:

- defining repeated simultaneous play is more subtle: Walras
- timing assumption changes not just payoffs, welfare, but *multiplicity* of equilibria

The Game Γ_0

Discretion is a game between private sector and central bank

For clarity, begin definition of game without central bank:

- assume interest rate process $\{r_t\}$ is i.i.d.
- call this game Γ_0

Game Γ_0:

- players
- payoffs
- information sets
- action spaces

Game Γ_0: Players and Payoffs

1. Firms indexed by $i \in [0, 1]$:

produce differentiated products; face Dixit-Stiglitz demand curves; have production function $y_t(i) = l_t(i)$; hire labor at wage rate w_t; payoff each period is profit:

$$\Pi_t(i) = p_t(i)y_t(i) - w_t l_t(i)$$

2. Households indexed by $j \in [0, 1]$:

supply labor $L_t(j)$; consume final good $C_t(j)$; borrow or lend a one-period nominal bond $B_t(j)$; payoff each period is utility flow:

$$u(C_s(j), L_s(j)) = \frac{C_s(j)^{1-\varphi} - 1}{1 - \varphi} - \chi_0 \frac{L_s(j)^{1+\chi}}{1 + \chi}$$

Note: there is a final good aggregator that is *not* a player of Γ_0

Game Γ_0: Information Sets

Individual households and firms are anonymous:

- only aggregate variables and aggregate outcomes are publicly observed

Information set of each firm i at time t is thus:

- history of aggregate outcomes: $\{C_s, L_s, P_s, r_s, w_s, \Pi_s\}$, $s < t$
- history of firm i's own actions

Information set of each household j at time t is thus:

- history of aggregate outcomes: $\{C_s, L_s, P_s, r_s, w_s, \Pi_s\}$, $s < t$
- history of household j's own actions

Aggregate Resource Constraints

In games of industry competition:

- Bertrand
- Cournot
- Stackelberg

Action spaces are just real numbers: e.g., price, quantity

In a macroeconomic game, there are aggregate resource constraints that must be respected, e.g.:

- total labor supplied by households must equal total labor demanded by firms
- total output supplied by firms must equal total consumption demanded by households
- money supplied by central bank must equal total money demanded by households (in game Γ_1)

Walrasian Auctioneer

To ensure that aggregate resource constraints are respected, we introduce a Walrasian auctioneer

- Instead of playing a price p_t, firms now play a price *schedule* $p_t(X_t)$, where X_t denotes aggregate variables realized at t
- this is just the usual NK assumption that firms take wages, interest rate, aggregates at time t as given

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- Instead of playing a consumption-labor pair (C_t, L_t), households play a joint *schedule* $(C_t(X_t), L_t(X_t))$
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Walrasian auctioneer then determines the equilibrium X_t that satisfies aggregate resource constraints

Game Γ_0: Action Spaces

1. Firms
   - set prices for two periods in Taylor contracts; must supply whatever output is demanded at posted price
   - firms in $[0, 1/2)$:
     for t odd, action space is set of measurable functions $p_t(X_t)$
     for t even, action space is trivial
   - firms in $[1/2, 1)$:
     for t even, action space is set of measurable functions $p_t(X_t)$
     for t odd, action space is trivial
2. Households
   - in each period, action space is set of measurable functions $(C_t(X_t), L_t(X_t))$

Game Γ_0: Action Spaces

Note:

- all firms *i* and households *j* play simultaneously in each period *t*
- Walrasian auctioneer clears markets, aggregate resource constraints

Also, do not confuse *action spaces* here with *strategies*:

- a *strategy* is a mapping from history h^t to the action space
- here, action spaces are functions of aggregate variables realized at *t*
- but strategies are unrestricted, may depend on arbitrary history of aggregate variables (until we impose Markovian restriction)

The Game Γ_1

Now, extend the game Γ_0 to include an optimizing central bank:

- interest rate r_t is set by central bank each period
- call this game Γ_1

First two sets of players (firms and households) are defined exactly as in Γ_0

Game Γ_1: Central Bank

3. Central bank:

sets one-period nominal interest rate r_t; payoff each period is given by average household welfare:

$$\int \frac{C_s(j)^{1-\varphi} - 1}{1-\varphi} - \chi_0 \frac{L_s(j)^{1+\chi}}{1+\chi} \, dj$$

Central bank's information set is the history of aggregate outcomes: $\{C_s, L_s, P_s, r_s, w_s, \Pi_s\}$, $s < t$

Note:

- central bank has no ability to commit to future actions (discretion)
- central bank is *monolithic,* while private sector is *atomistic*

Within-Period Timing of Play

Repeated Stackelberg play:

- each period divided into two halves
- first, central bank precommits to a value for r_t (or m_t)
- second, firms and households play simultaneously
- Walrasian auctioneer determines equilibrium
- note: one can drop the Walrasian auctioneer here if willing to ignore out-of-equilibrium play by positive μ of firms, households

Repeated simultaneous play:

- firms, households, and central bank all play simultaneously
- Walrasian auctioneer determines equilibrium
- note: Walrasian auctioneer is crucial, cannot be dropped (central bank is nonatomistic)

Game Γ_1: Action Spaces

In defining the game Γ_1, we assume repeated simultaneous play:

- firms i, households j, and central bank all play simultaneously in each period t
- action spaces of firms, households are same as in Γ_0
- for central bank, action space each period is set of measurable functions $r_t(X_t)$ (simultaneous play)
- Walrasian auctioneer clears markets, aggregate resource constraints

Again, do not confuse action spaces with strategies:

- strategies are unrestricted, may depend on arbitrary history of aggregate variables (until we impose Markovian restriction)

Why Assume Simultaneous Play?

Practical considerations/realism:

- Makes no difference whether monetary instrument is r_t or m_t
- Central banks monitor economic conditions continuously, adjust policy as needed

Theoretical considerations:

- Why treat central bank, private sector so asymmetrically?
- LQ literature (Svensson-Woodford 2003, 2004, Woodford 2003, Pearlman 1994, etc.) assumes simultaneous play
- Investigate sensitivity of multiple equilibria to within-period timing

Solving for Markov Perfect Equilibria

4 Solving for Markov Perfect Equilibria
- State Variables of the Game Γ_1
- Policymaker Bellman Equation
- Markov Perfect Equilibria of the Game Γ_1

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- distribution of household bond holdings, $B_{t-1}(j)$, $j \in [0, 1]$
- two measures of the distribution of inherited prices:

$$\int p_{t-1}(i)^{-1/\theta}\, di$$

and

$$\int p_{t-1}(i)^{-(1+\theta)/\theta}\, di$$

State Variables of the Game Γ_1

However, starting from symmetric initial conditions in period $t - 1$:

Proposition 1:

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We henceforth restrict defintion of game Γ_1 to case of symmetric initial conditions in period t_0

Policymaker Bellman Equation

$$V_t = \max_{\{r_t\}} \left\{ \int \frac{Y_t(j)^{1-\varphi}}{1-\varphi} - \chi_0 \frac{L_t(j)^{1+\chi}}{1+\chi} \, dj + \beta E_t V_{t+1} \right\}$$

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subject to:

$$\frac{L_t}{Y_t} = 2^{\theta} \frac{1 + x_t^{(1+\theta)/\theta}}{\left(1 + x_t^{1/\theta}\right)^{1+\theta}},$$

$$Y_t^{-\varphi}(1 + x_t^{1/\theta}) = \beta(1 + r_t)h_{1t},$$

$$2^{-\theta}\left(1+x_t^{1/\theta}\right)^{\theta}\left[Y_t^{1-\varphi} + \beta(1 + x_t^{1/\theta})h_{2t}\right] = (1+\theta)\chi_0\left[Y_t L_t^{\chi} + \beta\left(1+x_t^{1/\theta}\right)^{1+\theta} h_{3t}\right].$$

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where expectations of next period variables are given functions of this period's economic state: h_{1t}, h_{2t}, h_{3t} (discretion)

Markov Perfect Equilibria of the Game Γ_1

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\begin{aligned}
h_{1t} &= E_t Y_{t+1}^{-\varphi}\left(1 + x_{t+1}^{-1/\theta}\right) = h_1 \\
h_{2t} &= E_t \frac{Y_{t+1}^{1-\varphi}}{1 + x_{t+1}^{-1/\theta}} = h_2 \\
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Note: we will not write out how play evolves off of the equilibrium path, but simply assert that it agents will continue to play optimally (Phelan-Stachetti, 2001)

Solving for Markov Perfect Equilibria

Solve: $$V_t = \max_{\{r_t\}} \left\{ \frac{Y_t^{1-\varphi}}{1-\varphi} - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} + \beta E_t V_{t+1} \right\}$$

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where h_1, h_2, h_3 are exogenous constants.

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Finally, impose equilibrium conditions: $h_1 = E_t Y_{t+1}^{-\varphi}(1 + x_{t+1}^{-1/\theta})$, $h_2 = E_t \frac{Y_{t+1}^{1-\varphi}}{1+x_{t+1}^{-1/\theta}}$, $h_3 = E_t \frac{Y_{t+1} L_{t+1}^{\chi}}{\left(1+x_{t+1}^{-1/\theta}\right)^{1+\theta}}$.

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Note: there can still be multiplicity here, e.g. if h_1, h_2, h_3 are "bad"

Results

Proposition 6: The inflation rate π in any Markov Perfect Equilibrium of the game Γ_1 must satisfy the condition:

$$\frac{1+\beta\pi^{(1+\theta)/\theta}}{1+\beta\pi^{1/\theta}}\,\frac{1+\pi^{1/\theta}}{1+\pi^{(1+\theta)/\theta}}\times$$

$$\left\{1-\frac{(\pi-1)\left[1+\chi-(1-\varphi)\frac{1+\beta\pi^{(1+\theta)/\theta}}{1+\beta\pi^{1/\theta}}\right]}{(\pi-1)\left[1-(1-\varphi)\frac{1+\beta\pi^{(1+\theta)/\theta}}{1+\beta\pi^{1/\theta}}\right]+(1+\pi^{(1+\theta)/\theta})\left[1-\frac{1}{1+\theta}\frac{1+\beta\pi^{(1+\theta)/\theta}}{1+\beta\pi^{1/\theta}}\right]}\right\}=\frac{1}{1+\theta} \qquad (*)$$

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Proposition 7: Let $\varphi = 1$, $\chi = 0$, and $\beta > \max\{1/2, 1/(1+2\theta)\}$. *Then there is precisely one value of π that satisfies equation $(*)$.*

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Note:

- $\varphi = 1$, $\chi = 0$ are not special, but simplify algebra in proofs
- there is a unique equilibrium for wide range of parameters
- confirmed by extensive numerical simulation in Matlab

Conclusions

- There are two definitions of “discretion” in the literature
- These definitions differ in terms of within-period timing of play
- Within-period timing makes a *huge* difference
- In the New Keynesian model with repeated Stackelberg play, there are multiple equilibria (King-Wolman, 2004)
- In the New Keyneisan model with repeated simultaneous play, there is a unique equilibrium (this paper)
- Open questions: other NK models, models with a (nondegenerate) state variable