Implications of Labor Market Frictions for Risk Aversion and Risk Premia

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Society for Economic Dynamics Meetings, Seoul
June 28, 2013

Coefficient of Relative Risk Aversion

Suppose a household has preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta\, l_t$$

What is the household's coefficient of relative risk aversion?

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Answer: 0

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What is the household's coefficient of relative risk aversion?

Answer: $\dfrac{1}{\frac{1}{\gamma} + \frac{1}{\chi}}$

Empirical Relevance of the Labor Margin

Imbens, Rubin, and Sacerdote (2001):

- Individuals who win a lottery prize reduce labor supply by $.11 for every $1 won (note: spouse may also reduce labor supply)

Coile and Levine (2009):

- Older individuals are 7% less likely to retire in a given year after a 30% fall in stock market

Coronado and Perozek (2003):

- Individuals who held more stocks in late 1990s retired 7 months earlier

Large literature estimating wealth effects on labor supply (e.g., Pencavel 1986)

Frictional Labor Markets

Perfectly rigid labor market:

- Arrow (1964), Pratt (1965), Epstein-Zin (1989), etc.

Perfectly flexible labor market:

- Swanson (2012)

This paper:

- Frictional labor markets

A Household

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \big[U(c_\tau) - V(l_\tau + u_\tau) \big],$$

Flow budget constraint:

$$a_{\tau+1} = (1 + r_\tau) a_\tau + w_\tau l_\tau + d_\tau - c_\tau,$$

No-Ponzi condition:

$$\lim_{T\to\infty} \prod_{\tau=t}^{T} (1 + r_{\tau+1})^{-1} a_{T+1} \geq 0,$$

$\{w_\tau, r_\tau, d_\tau\}$ are exogenous processes, governed by θ_τ

Labor market search: $\quad l_{\tau+1} = (1-s) l_\tau + f(\theta_\tau) u_\tau$

The Value Function

State variables of the household's problem are $(a_t, l_t; \theta_t)$.

Let:

$$c_t^* \equiv c^*(a_t, l_t; \theta_t),$$

$$u_t^* \equiv u^*(a_t, l_t; \theta_t).$$

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Value function, Bellman equation:

$$W(a_t, l_t; \theta_t) = U(c_t^*) - V(l_t + u_t) + \beta E_t W(a_{t+1}^*, l_{t+1}^*; \theta_{t+1}),$$

where:

$$a_{t+1}^* \equiv (1 + r_t)a_t + w_t l_t + d_t - c_t^*,$$
$$l_{t+1}^* \equiv (1-s)l_t + f(\theta_t)u_t^*.$$

Technical Conditions

Assumption 1. *The function $U(c_t)$ is increasing, twice-differentiable, and strictly concave, and $V(l_t)$ is increasing, twice-differentiable, and strictly convex.*

Assumption 2. *A solution $W : X \to \mathbb{R}$ to the household's generalized Bellman equation exists and is unique, continuous, and concave.*

Assumption 3. *For any $(a_t, l_t; \theta_t) \in X$, the household's optimal choice (c_t^*, u_t^*) exists, is unique, and lies in the interior of $\Gamma(a_t; \theta_t)$.*

Assumption 4. *For any $(a_t, l_t; \theta_t)$ in the interior of X, the second derivative of W with respect to its first argument, $W_{11}(a_t, l_t; \theta_t)$, exists.*

Assumptions about the Economic Environment

Assumption 5. *The household is* infinitesimal*.*

Assumption 6. *The household is* representative*.*

Assumption 7. *The model has a* nonstochastic steady state*,* $x_t = x_{t+k}$ *for* $k = 1, 2, \ldots,$ *and* $x \in \{c, l, a, w, r, d, \theta\}$*.*

Assumption 7′. *The model has a* balanced growth path *that can be renormalized to a nonstochastic steady state after a suitable change of variables.*

Arrow-Pratt in a Static One-Good Model

Compare:

$$E\,u(c + \sigma\varepsilon) \quad \text{vs.} \quad u(c - \mu)$$

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Arrow-Pratt coefficient of absolute risk aversion:

$$\lim_{\sigma \to 0} 2\mu(\sigma)/\sigma^2$$

Arrow-Pratt in a Dynamic Model

Consider a one-shot gamble in period t:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma\varepsilon_{t+1},$$

vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu.$$

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Definition 1. *The household's coefficient of absolute risk aversion at* $(a_t, l_t; \theta_t)$ *is given by* $R^a(a_t, l_t; \theta_t) = \lim_{\sigma\to 0} 2\mu(\sigma)/\sigma^2$.

Coefficient of Absolute Risk Aversion

Proposition 1. *The household's coefficient of absolute risk aversion at* $(a_t, l_t; \theta_t)$*, denoted* $R^a(a_t, l_t; \theta_t)$*, satisfies*

$$\frac{-E_t W_{11}(a^*_{t+1}, l^*_{t+1}; \theta_{t+1})}{E_t W_1(a^*_{t+1}, l^*_{t+1}; \theta_{t+1})}.$$

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Evaluated at the nonstochastic steady state, this simplifies to:

$$R^a(a, l; \theta) = \frac{-W_{11}(a, l; \theta)}{W_1(a, l; \theta)}$$

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Folk wisdom: Constantinides (1990), Farmer (1990), Campbell-Cochrane (1999), Boldrin-Christiano-Fisher (1997, 2001), Flavin-Nakagawa (2008)

Solve for W_1 and W_{11}

Benveniste-Scheinkman:

$$W_1(a_t, l_t; \theta_t) = (1 + r_t)\, U'(c_t^*). \qquad (*)$$

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Differentiate $(*)$ to get:

$$W_{11}(a_t, l_t; \theta_t) \;=\; (1 + r_t) U''(c_t^*) \frac{\partial c_t^*}{\partial a_t}\,.$$

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Consumption Euler equation:

$$\frac{\partial c_t^*}{\partial a_t} \; = \; \frac{\partial c_{t+1}^*}{\partial a_t} \; = \; \frac{\partial c_{t+k}^*}{\partial a_t}\,, \quad k = 1, 2, \ldots$$

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Labor search Euler equation:

$$\frac{\partial l_{t+k}^*}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\theta)}{s + f(\theta)} \left[1 - \left(1 - s - f(\theta)\right)^k\right] \frac{\partial c_t^*}{\partial a_t}\,.$$

Solve for W_1 and W_{11}

Budget constraint:

$$\frac{\partial c_t^*}{\partial a_t} = \frac{r}{1 + w\,\frac{\gamma}{\chi}\,\frac{l+u}{c}\,\frac{f(\theta)}{r+s+f(\theta)}}.$$

Solve for W_1 and W_{11}

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$$\frac{\partial c_t^*}{\partial a_t} = \frac{r}{1 + w\,\frac{\gamma}{\chi}\,\frac{l+u}{c}\,\frac{f(\theta)}{r+s+f(\theta)}}.$$

Proposition 2. *The household's coefficient of absolute risk aversion in Proposition 1, evaluated at steady state, satisfies:*

$$R^a(a;\theta) = \frac{-U''(c)}{U'(c)}\,\frac{r}{1 + w\frac{\gamma}{\chi}\,\frac{l+u}{c}\,\frac{f(\theta)}{r+s+f(\theta)}}.$$

Relative Risk Aversion

Compare: $a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma A_t \varepsilon_{t+1}$

vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu A_t.$$

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vs.

$$a_{t+1} = (1+r_t)a_t + w_t l_t + d_t - c_t - \mu A_t .$$

Definition 2. *The coefficient of relative risk aversion, $R^c(a_t;\theta_t) \equiv A_t^c R^a(a_t;\theta_t)$, where A_t^c denotes the present discounted value of household consumption.*

At steady state, $A^c = c/r$, and

$$R^c(a;\theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + w \dfrac{\gamma}{\chi} \dfrac{l+u}{c} \dfrac{f(\theta)}{r+s+f(\theta)}} .$$

Numerical Asset Pricing Example

Household period utility function:

$$\frac{c_t^{1-\gamma}}{1-\gamma} - \eta \frac{(l_t + u_t)^{1+\chi}}{1+\chi}, \qquad \gamma = 200$$

Economy is a very simple, standard RBC model:

- Competitive firms
- Cobb-Douglas production, $y_t = Z_t k_t^{1-\alpha} l_t^{\alpha}$
- AR(1) technology, $\log Z_{t+1} = \rho_z \log Z_t + \varepsilon_t$
- Capital accumulation, $k_{t+1} = (1-\delta)k_t + y_t - c_t$
- Equity is a consumption claim
- Equity premium is expected excess return,

$$\psi_t = \frac{E_t(C_{t+1} + p_{t+1})}{p_t} - (1 + r_t^f)$$

Figure 1: Risk Aversion and Equity Premium vs. χ

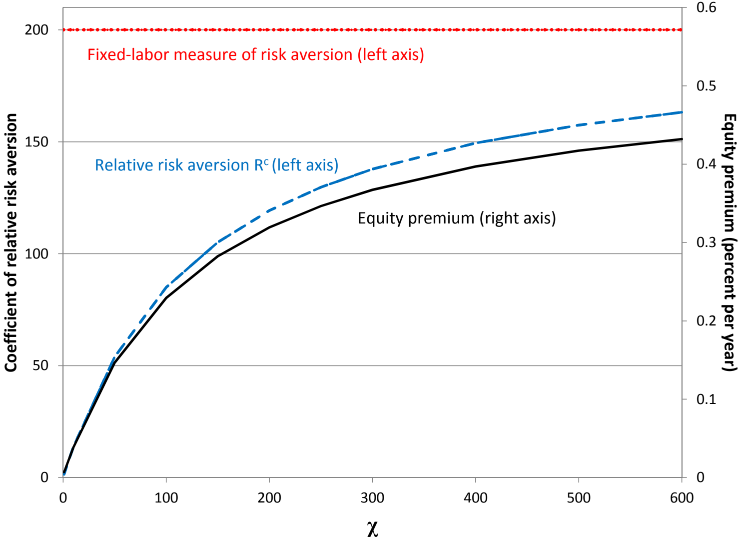


Figure 2: Risk Aversion and Equity Premium vs. γ

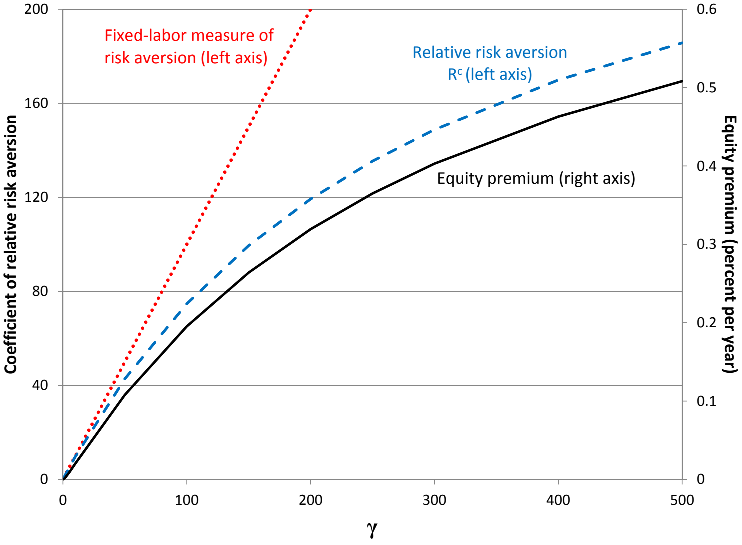
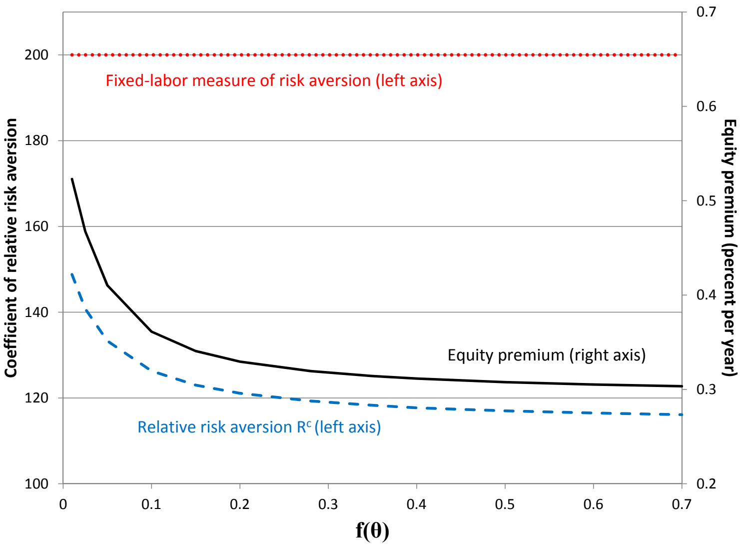


Figure 3: Risk Aversion and Equity Premium vs. $f(\theta)$



Risk Aversion Is Higher in Recessions

Proposition 3. *Given Assumptions 1–8 and fixed values for the parameters s, β, γ, and χ, $R^c(a, l; \theta)$ is decreasing in l/u.*

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Proof:

$$R^c(a;\theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + w\,\frac{\gamma}{\chi}\,\frac{l+u}{c}\,\frac{f(\theta)}{r+s+f(\theta)}}.$$

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Using $sl = f(\theta)u$,

$$R^c(a;\theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \frac{\gamma}{\chi}\,\frac{wl}{c}\,\frac{s(1+l/u)}{r+s(1+l/u)}}.$$

Risk Aversion Higher in More Frictional Labor Markets

Proposition 4. *Let* $f_1, f_2 : \Theta \to \mathbb{R}$*. Given Assumptions 1–8, let* $(a_1, l_1; \theta_1)$ *and* $(a_2, l_2; \theta_2)$ *denote corresponding steady-state values of* $(a_t, l_t; \theta_t)$*. If* $f_1(\theta_1) < f_2(\theta_2)$*, then* $R_1^c(a_1, l_1; \theta_1) > R_2^c(a_2, l_2; \theta_2)$*.*

Risk Aversion Higher in More Frictional Labor Markets

Proposition 4. *Let $f_1, f_2 : \Theta \to \mathbb{R}$. Given Assumptions 1–8, let $(a_1, l_1; \theta_1)$ and $(a_2, l_2; \theta_2)$ denote corresponding steady-state values of $(a_t, l_t; \theta_t)$. If $f_1(\theta_1) < f_2(\theta_2)$, then $R_1^c(a_1, l_1; \theta_1) > R_2^c(a_2, l_2; \theta_2)$.*

Proof:

$$R^c(a;\theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \frac{\gamma}{\chi} \frac{wl}{c} \frac{s + f(\theta)}{r + s + f(\theta)}}.$$

Risk Aversion Higher for Less Employable Households

Two types of households:

- Measure 1 of type 1 households
- Measure 0 ot type 2 households
- Type 1 households are more employable: $f_1(\theta) > f_2(\theta)$

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Then Proposition 4 implies $R_2^c(a, l; \theta) > R_1^c(a, l; \theta)$.

Table 1: International Comparison

	s	$f(\theta)$	percentage of households owning equities	percentage of households owning risky financial assets	share of household portfolios in currency and deposits
United States	.019	.282	48.9	49.2	12.4
United Kingdom	.009	.056	31.5	32.4	26.0
Germany	.006	.035	18.9	25.1	33.9
France	.007	.033	–	–	29.1
Spain	.012	.020	–	–	38.1
Italy	.004	.013	18.9	22.1	27.9

Table 2: International Comparison

	s	$f(\theta)$	$\frac{s+f(\theta)}{r+s+f(\theta)}$	Relative Risk Aversion R^c $\gamma = 2$ $\chi = 1.5$	$\gamma = 5$ $\chi = 0.5$	$\gamma = 10$ $\chi = 2.5$	$\gamma = 20$ $\chi = 10$
Theoretical labor market benchmarks:							
perfect rigidity	0	0	0	2	5	10	20
near-perfect flexibility	1	1	.998	0.86	0.46	2.00	6.68
International comparison (based on Hobijn and Şahin, 2007):							
United States	.019	.282	.987	0.86	0.46	2.02	6.73
United Kingdom	.009	.056	.942	0.89	0.48	2.10	6.93
Germany	.006	.035	.911	0.90	0.49	2.15	7.09
France	.007	.033	.909	0.90	0.50	2.16	7.10
Spain	.012	.020	.889	0.92	0.51	2.20	7.20
Italy	.004	.013	.810	0.96	0.55	2.36	7.64
Business cycle variation (based on Shimer, 2012):							
United States, expansion	.017	.35	.989	0.86	0.46	2.02	6.71
United States, recession	.022	.20	.982	0.87	0.46	2.03	6.75

Table 3: Higher Costs of Labor Market Frictions

	s	$f(\theta)$	$\frac{s+f(\theta)}{r+s+f(\theta)}$	Relative Risk Aversion R^c $\gamma = 2$ $\chi = 1.5$	 $\gamma = 5$ $\chi = 0.5$	 $\gamma = 10$ $\chi = 2.5$	 $\gamma = 20$ $\chi = 10$
$r = .004$:							
United States	.006	.282	.986	0.86	0.46	2.02	6.73
Italy	.004	.013	.810	0.96	0.55	2.36	7.64
$r = .008$:							
United States	.006	.282	.973	0.87	0.47	2.04	6.79
Italy	.004	.013	.680	1.05	0.64	2.69	8.47
$r = .012$:							
United States	.006	.282	.960	0.88	0.47	2.07	6.85
Italy	.004	.013	.586	1.12	0.73	2.99	9.21

Conclusions

General conclusions:

- A flexible labor margin affects risk aversion
- Risk premia are closely related to risk aversion

Implications of labor market frictions:

- Risk aversion is higher in recessions
- Risk aversion is higher in more frictional labor markets
- Risk aversion is higher for households that are less employable

Quantitative findings:

- Low discount rate $\Rightarrow$ effects of labor market frictions are small
- Risk aversion formulas in Swanson (2012) a good approximation
- Quantitative effects of frictions larger if frictions are more costly