Implications of Labor Market Frictions for Risk Aversion and Risk Premia

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Coefficient of Relative Risk Aversion

Suppose a household has preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta\, l_t$$

What is the household's coefficient of relative risk aversion?

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Answer: 0

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What is the household's coefficient of relative risk aversion?

Answer: $\dfrac{1}{\frac{1}{\gamma} + \frac{1}{\chi}}$

Empirical Relevance of the Labor Margin

Imbens, Rubin, and Sacerdote (2001):

- Individuals who win a lottery prize reduce labor supply by $.11 for every $1 won (note: spouse may also reduce labor supply)

Coile and Levine (2009):

- Older individuals are 7% less likely to retire in a given year after a 30% fall in stock market

Coronado and Perozek (2003):

- Individuals who held more stocks in late 1990s retired 7 months earlier

Large literature estimating wealth effects on labor supply (e.g., Pencavel 1986)

Frictional Labor Markets

Perfectly rigid labor market:

- Arrow (1964), Pratt (1965), Epstein-Zin (1989), etc.

Perfectly flexible labor market:

- Swanson (2012, 2013)

This paper:

- Frictional labor markets

A Household

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(c_\tau) - V(l_\tau + u_\tau) \right],$$

Flow budget constraint:

$$a_{\tau+1} = (1 + r_\tau) a_\tau + w_\tau l_\tau + d_\tau - c_\tau,$$

No-Ponzi condition:

$$\lim_{T\to\infty} \prod_{\tau=t}^{T} (1 + r_{\tau+1})^{-1} a_{T+1} \geq 0,$$

$\{w_\tau, r_\tau, d_\tau\}$ are exogenous processes, governed by Θ_τ

Labor market search: $\quad l_{\tau+1} = (1-s) l_\tau + f(\Theta_\tau) u_\tau$

The Value Function

State variables of the household's problem are $(a_t, l_t; \Theta_t)$.

Let:

$$c_t^* \equiv c^*(a_t, l_t; \Theta_t),$$

$$u_t^* \equiv u^*(a_t, l_t; \Theta_t).$$

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Value function, Bellman equation:

$$\mathbb{V}(a_t, l_t; \Theta_t) = U(c_t^*) - V(l_t + u_t^*) + \beta E_t \mathbb{V}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}),$$

where:

$$a_{t+1}^* \equiv (1 + r_t)a_t + w_t l_t + d_t - c_t^*,$$
$$l_{t+1}^* \equiv (1-s)l_t + f(\Theta_t)u_t^*.$$

Technical Conditions

Assumption 1. *The function $U(c_t)$ is increasing, twice-differentiable, and strictly concave, and $V(l_t)$ is increasing, twice-differentiable, and strictly convex.*

Assumption 2. *A solution $\mathbb{V} : X \to \mathbb{R}$ to the household's generalized Bellman equation exists and is unique, continuous, and concave.*

Assumption 3. *For any $(a_t, l_t; \Theta_t) \in X$, the household's optimal choice (c_t^*, u_t^*) exists, is unique, and lies in the interior of $\Gamma(a_t, l_t; \Theta_t)$.*

Assumption 4. *For any $(a_t, l_t; \Theta_t)$ in the interior of X, the second derivative of $\mathbb{V}$ with respect to its first argument, $\mathbb{V}_{11}(a_t, l_t; \Theta_t)$, exists.*

Assumptions about the Economic Environment

Assumption 5. *The household is* infinitesimal*.*

Assumption 6. *The household is* representative*.*

Assumption 7. *The model has a* nonstochastic steady state*,* $x_t = x_{t+k}$ *for* $k = 1, 2, \ldots,$ *and* $x \in \{c, u, l, a, w, r, d, \Theta\}$*.*

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Assumption 7′. *The model has a* balanced growth path *that can be renormalized to a nonstochastic steady state after a suitable change of variables.*

Arrow-Pratt in a Static One-Good Model (Review)

Compare:

$$E\,u(c + \sigma\varepsilon) \quad \text{vs.} \quad u(c - \mu)$$

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$$u(c-\mu) \;\approx\; u(c) - \mu u'(c),$$

$$E\,u(c+\sigma\varepsilon) \;\approx\; u(c) + u'(c)\sigma E[\varepsilon] + \frac{1}{2}u''(c)\sigma^2 E[\varepsilon^2],$$

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$$\mu = \frac{-u''(c)}{u'(c)}\,\frac{\sigma^2}{2}.$$

Coefficient of absolute risk aversion is defined to be:

$$\lim_{\sigma\to 0} 2\mu(\sigma)/\sigma^2 = \frac{-u''(c)}{u'(c)}.$$

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Consider a one-shot gamble in period t:

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Note $(*)$ is equivalent to gamble over asset returns:

$$a_{t+1} = (1 + r_t + \sigma\tilde{\varepsilon}_{t+1})a_t + w_t l_t + d_t - c_t.$$

or income:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + (d_t + \sigma\varepsilon_{t+1}) - c_t,$$

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Welfare loss from μ:

$$\mathbb{V}_1(a_t, l_t; \Theta_t)\,\frac{\mu}{(1 + r_t)}$$

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$$\beta E_t \mathbb{V}_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})\, \mu.$$

Loss from σ:

$$\beta E_t \mathbb{V}_{11}(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1}) \frac{\sigma^2}{2}.$$

Coefficient of Absolute Risk Aversion

Definition 1. *The household's coefficient of absolute risk aversion at* $(a_t, l_t; \Theta_t)$ *is given by* $R^a(a_t, l_t; \Theta_t) = \lim_{\sigma \to 0} 2\mu(\sigma)/\sigma^2$.

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Proposition 1. *The household's coefficient of absolute risk aversion at* $(a_t, l_t; \Theta_t)$ *is well-defined and satisfies*

$$R^a(a_t, l_t; \Theta_t) \; = \; \frac{-E_t \mathbb{V}_{11}(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}{E_t \mathbb{V}_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})} \, .$$

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Folk wisdom: Constantinides (1990), Farmer (1990), Campbell-Cochrane (1999), Boldrin-Christiano-Fisher (1997, 2001), Flavin-Nakagawa (2008)

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Evaluated at the nonstochastic steady state, this simplifies to:

$$R^a(a, l; \Theta) \; = \; \frac{-\mathbb{V}_{11}(a, l; \Theta)}{\mathbb{V}_1(a, l; \Theta)}$$

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Solve for $\mathbb{V}_1$ and $\mathbb{V}_{11}$

Household preferences:

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Differentiate $(*)$ to get:

$$\mathbb{V}_{11}(a_t, l_t; \Theta_t) \;=\; (1 + r_t) U''(c_t^*) \frac{\partial c_t^*}{\partial a_t} .$$

Solve for $\partial c_t^* / \partial a_t$

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Consumption Euler equation:

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implies, at steady state:

$$\frac{\partial c_t^*}{\partial a_t} \; = \; E_t \frac{\partial c_{t+1}^*}{\partial a_t} \; = \; E_t \frac{\partial c_{t+k}^*}{\partial a_t}, \quad k = 1, 2, \ldots$$

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Household's budget constraint, no-Ponzi condition imply:

$$\sum_{k=0}^{\infty} \frac{1}{(1+r)^k}\, E_t \left[\frac{\partial c_{t+k}^*}{\partial a_t} - w \frac{\partial l_{t+k}^*}{\partial a_t} \right] \;=\; 1 + r\,.$$

Solve for $\partial l^*_{t+k}/\partial a_t$

Labor search Euler equation:

$$\frac{V'(l_t + u_t^*)}{f(\Theta_t)} = \beta E_t \Big[w_{t+1} U'(c^*_{t+1}) - V'(l^*_{t+1} + u^*_{t+1}) + (1-s)\frac{V'(l^*_{t+1} + u^*_{t+1})}{f(\Theta_{t+1})} \Big]$$

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and transition equation

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imply, at steady state:

$$E_t\frac{\partial l^*_{t+k}}{\partial a_t} = -\frac{\gamma}{\chi}\frac{l+u}{c}\frac{f(\Theta)}{s+f(\Theta)}\left[1-(1-s-f(\Theta))^k\right]\frac{\partial c^*_t}{\partial a_t}.$$

where $\gamma \equiv -cU''(c)/U'(c)$, $\chi \equiv (l+u)V''(l+u)/V'(l+u)$

Solve for $\partial c_t^* / \partial a_t$

Household's budget constraint, no-Ponzi condition:

$$\sum_{k=0}^{\infty} \frac{1}{(1+r)^k} E_t \left[\frac{\partial c_{t+k}^*}{\partial a_t} - w \frac{\partial l_{t+k}^*}{\partial a_t} \right] = 1 + r \,,$$

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Labor Euler equation:

$$E_t \frac{\partial l_{t+k}^*}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l+u}{c} \frac{f(\Theta)}{s + f(\Theta)} \left[1 - \left(1 - s - f(\Theta) \right)^k \right] \frac{\partial c_t^*}{\partial a_t} \,,$$

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Solution is a "modified Golden Rule":

$$\frac{\partial c_t^*}{\partial a_t} = \frac{r}{1 + w \, \frac{\gamma}{\chi} \, \frac{l+u}{c} \, \frac{f(\Theta)}{r + s + f(\Theta)}} \, .$$

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Proposition 2. *Given Assumptions 1–7, the household's coefficient of absolute risk aversion, $R^a(a_t, l_t; \Theta_t)$, evaluated at steady state, satisfies*

$$R^a(a, l; \Theta) \; = \; \frac{-U''(c)}{U'(c)}\, \frac{r}{1 + w\, \frac{\gamma}{\chi}\, \frac{l + u}{c}\, \frac{f(\Theta)}{r + s + f(\Theta)}}.$$

Relative Risk Aversion

Compare: $a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma A_t \varepsilon_{t+1}$

vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu A_t.$$

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Definition 2. *The households' coefficient of relative risk aversion, $R^c(a_t, l_t; \Theta_t) \equiv A_t R^a(a_t, l_t; \Theta_t)$, where A_t denotes the household's financial assets plus present discounted value of labor income.*

At steady state, $A = c/r$, and

$$R^c(a; \Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + w \dfrac{\gamma}{\chi} \dfrac{l + u}{c} \dfrac{f(\Theta)}{r + s + f(\Theta)}}.$$

Numerical Example

Household period utility function:

$$\frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{(l_t + u_t)^{1+\chi}}{1+\chi}$$

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Economy is a simple RBC model with labor market frictions:

- Competitive firms,
- Cobb-Douglas production functions, $y_t = Z_t k_t^{1-\alpha} l_t^{\alpha}$
- AR(1) technology, $\log Z_{t+1} = \rho_z \log Z_t + \varepsilon_t$
- Capital accumulation, $k_{t+1} = (1-\delta)k_t + y_t - c_t$
- Labor market frictions, $l_{t+1} = (1-s)l_t + h_t$

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Labor market search:

- Cobb-Douglas matching function, $h_t = \mu u_t^{1-\eta} v_t^{\eta}$
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Equity security:

- Equity is a consumption claim
- Equity premium is expected excess return,

$$\psi_t \equiv \frac{E_t(C_{t+1} + p_{t+1})}{p_t} - (1 + r_t^f)$$

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Baseline calibration:

- Production: $\alpha = 0.7$, $\delta = .0028$, $\rho_z = 0.98$, $\sigma_\varepsilon = .005$
- Matching: $s = .02$, $\eta = 0.5$, $v/u = 0.6$, $f(\Theta) = 0.28$
- Preferences: $\beta = .996$, $\gamma = 200$, $\chi = 200$, $l + u = 0.3$

Figure 1: Risk Aversion and Equity Premium vs. χ

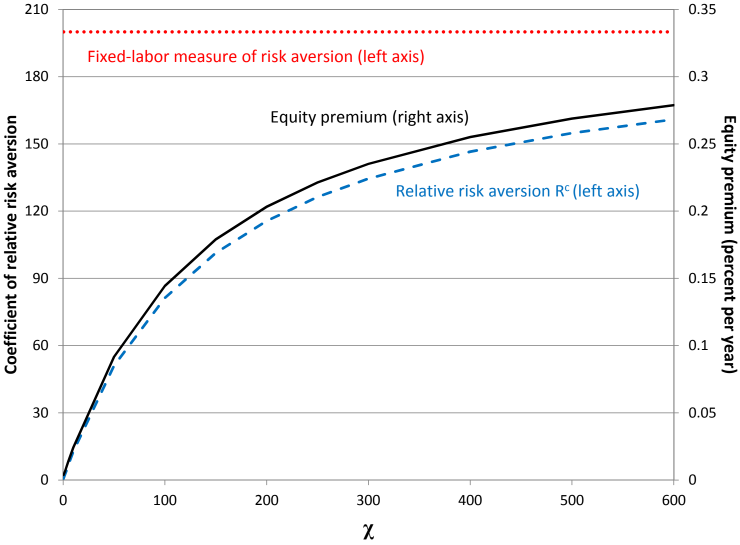


Figure 2: Risk Aversion and Equity Premium vs. γ

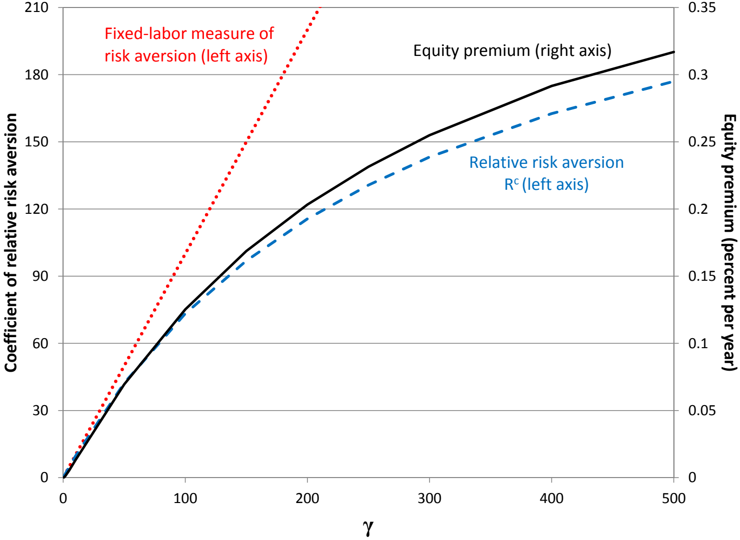
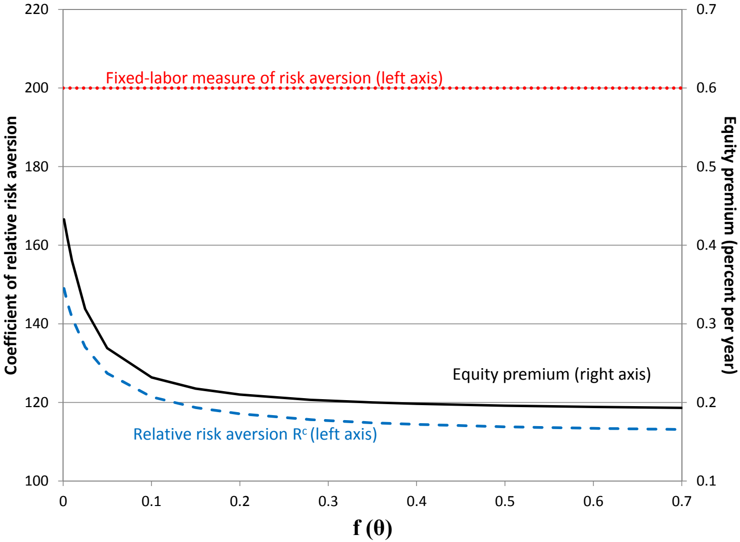


Figure 3: Risk Aversion and Equity Premium vs. $f(\Theta)$



Risk Aversion Is Higher in Recessions

Proposition 3. *Given Assumptions 1–8 and fixed values for the parameters* s, β, γ, *and* χ, $R^c(a, l; \Theta)$ *is decreasing in* l/u.

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Proof:

$$R^c(a, l; \Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + w \dfrac{\gamma}{\chi} \dfrac{l+u}{c} \dfrac{f(\Theta)}{r + s + f(\Theta)}}.$$

Using $sl = f(\Theta)u$,

$$R^c(a, l; \Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \dfrac{\gamma}{\chi} \dfrac{wl}{c} \dfrac{s(1 + l/u)}{r + s(1 + l/u)}}.$$

Risk Aversion Higher in More Frictional Labor Markets

Proposition 4. *Let* $f_1, f_2 : \Omega_\Theta \to [0, 1]$. *Given Assumptions 1–8 and fixed values for the parameters s,* β, γ, *and* χ, *let* $(a_1, l_1; \Theta_1)$ *and* $(a_2, l_2; \Theta_2)$ *denote corresponding steady-state values of* $(a_t, l_t; \Theta_t)$. *If* $f_1(\Theta_1) < f_2(\Theta_2)$, *then* $R_1^c(a_1, l_1; \Theta_1) > R_2^c(a_2, l_2; \Theta_2)$.

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Proof:

$$R^c(a, l; \Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \frac{\gamma}{\chi} \frac{wl}{c} \frac{s + f(\Theta)}{r + s + f(\Theta)}}$$

is decreasing in $f(\Theta)$.

Risk Aversion Higher for Less Employable Households

Two types of households:

- Measure 1 of type 1 households
- Measure 0 ot type 2 households
- Type 1 households are more employable: $f_1(\Theta) > f_2(\Theta)$

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Then Proposition 4 implies $R_2^c(a_2, l_2; \Theta) > R_1^c(a_1, l_1; \Theta)$.

Table 1: International Comparison

	s	$f(\Theta)$	percentage of households owning equities	percentage of households owning risky financial assets	share of household portfolios in currency and deposits
United States	.019	.282	48.9	49.2	12.4
United Kingdom	.009	.056	31.5	32.4	26.0
Germany	.006	.035	18.9	25.1	33.9
France	.007	.033	–	–	29.1
Spain	.012	.020	–	–	38.1
Italy	.004	.013	18.9	22.1	27.9

Table 2: International Comparison

	s	$f(\Theta)$	$\frac{s+f(\Theta)}{r+s+f(\Theta)}$	Relative Risk Aversion R^c $\gamma = 2$ $\chi = 1.5$	 $\gamma = 5$ $\chi = 0.5$	 $\gamma = 10$ $\chi = 2.5$	 $\gamma = 20$ $\chi = 10$
Theoretical labor market benchmarks:							
perfect rigidity	0	0	0	2	5	10	20
near-perfect flexibility	1	1	.998	0.86	0.46	2.00	6.68
International comparison, $r = .004$:							
United States	.019	.282	.987	0.86	0.46	2.02	6.73
United Kingdom	.009	.056	.942	0.89	0.48	2.10	6.93
Germany	.006	.035	.911	0.90	0.49	2.15	7.09
France	.007	.033	.909	0.90	0.50	2.16	7.10
Spain	.012	.020	.889	0.92	0.51	2.20	7.20
Italy	.004	.013	.810	0.96	0.55	2.36	7.64

Table 2: International Comparison

				Relative Risk Aversion R^c			
				$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
	s	$f(\Theta)$	$\frac{s+f(\Theta)}{r+s+f(\Theta)}$	$\chi = 1.5$	$\chi = 0.5$	$\chi = 2.5$	$\chi = 10$
Theoretical labor market benchmarks:							
perfect rigidity	0	0	0	2	5	10	20
near-perfect flexibility	1	1	.998	0.86	0.46	2.00	6.68
International comparison, $r = .0083$:							
United States	.019	.282	.973	0.87	0.47	2.04	6.79
United Kingdom	.009	.056	.887	0.92	0.51	2.20	7.21
Germany	.006	.035	.832	0.95	0.54	2.31	7.51
France	.007	.033	.828	0.95	0.54	2.32	7.53
Spain	.012	.020	.794	0.97	0.56	2.40	7.73
Italy	.004	.013	.672	1.05	0.65	2.71	8.53

Table 2: International Comparison

	s	$f(\Theta)$	$\frac{s+f(\Theta)}{r+s+f(\Theta)}$	Relative Risk Aversion R^c $\gamma=2$ $\chi=1.5$	$\gamma=5$ $\chi=0.5$	$\gamma=10$ $\chi=2.5$	$\gamma=20$ $\chi=10$
Theoretical labor market benchmarks:							
perfect rigidity	0	0	0	2	5	10	20
near-perfect flexibility	1	1	.998	0.86	0.46	2.00	6.68
International comparison, $r=.0167$:							
United States	.019	.282	.947	0.88	0.48	2.09	6.91
United Kingdom	.009	.056	.796	0.97	0.56	2.39	7.72
Germany	.006	.035	.711	1.03	0.62	2.60	8.26
France	.007	.033	.705	1.03	0.62	2.62	8.30
Spain	.012	.020	.657	1.07	0.66	2.76	8.64
Italy	.004	.013	.504	1.20	0.83	3.31	9.96

Table 3: Cyclical Variation in Risk Aversion

				Relative Risk Aversion R^c			
				$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
	s	$f(\Theta)$	$\frac{s+f(\Theta)}{r+s+f(\Theta)}$	$\chi = 1.5$	$\chi = 0.5$	$\chi = 2.5$	$\chi = 10$
$r = .004$:							
United States, expansion	.017	.35	.989	0.86	0.46	2.02	6.71
United States, recession	.022	.20	.982	0.87	0.46	2.03	6.75
$r = .0083$:							
United States, expansion	.017	.35	.978	0.87	0.46	2.04	6.77
United States, recession	.022	.20	.964	0.88	0.47	2.06	6.83
$r = .0167$:							
United States, expansion	.017	.35	.956	0.88	0.47	2.07	6.87
United States, recession	.022	.20	.930	0.89	0.49	2.12	6.99

Conclusions

General conclusions:

- A flexible labor margin affects risk aversion
- Risk premia are closely related to risk aversion

Implications of labor market frictions:

- Risk aversion is higher in recessions
- Risk aversion is higher in more frictional labor markets
- Risk aversion is higher for households that are less employable

Quantitative findings:

- Low discount rate $\Rightarrow$ effects of labor market frictions are small
- Risk aversion formulas in Swanson (2012) a good approximation
- Quantitative effects of frictions can be substantial if discount rate is high (frictions are more costly)